Optimal Thermal Insulation Thickness in Isolated Air-Conditioned Buildings and Economic Analysis

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Abstract

The removal building heat load and electrical power consumption by air conditioning system are proportional to the outside conditions and solar radiation intensity. Building construction materials has substantial effects on the transmission heat through outer walls, ceiling and glazing windows. Good thermal isolation for buildings is important to reduce the transmitted heat and consumed power. The buildings models are constructed from common materials with 0 - 16 cm of thermal insulation thickness in the outer walls and ceilings, and double-layers glazing windows. The building heat loads were calculated for two types of walls and ceiling with and without thermal insulation. The cooling load temperature difference method, *CLTD*, was used to estimate the building heat load during a 24-hour each day throughout spring, summer, autumn and winter seasons. The annual cooling degree-day, *CDD* was used to estimate the optimal thermal insulation thickness and payback period with including the solar radiation effect on the outer walls surfaces. The average saved energy percentage in summer, spring, autumn and winter are 35.5%, 32.8%, 33.2% and 30.7% respectively, and average yearly saved energy is about of 33.5%. The optimal thermal insulation thickness was obtained between 7 - 12 cm and payback period of 20 - 30 month for some Egyptian Cities according to the Latitude and annual degree-days.

Keywords

Building Heat Load, Cooling Load Temperature Difference, Energy Saving, Power Consumption, Annual Cooling Degree-Day, Optimal Thermal Insulation Thickness, Payback Period

1. Introduction

The buildings heat load is proportional to the transmission heat through outside walls, ceiling, and glazing windows. The transmission heat load is added to the internal building heat load in summer season and subtracted from it in winter season. The thermal insulation is an effective tool to achieve the minimum heat transmission through walls. The transmission heat is a function of the thermal resistance of composite wall materials which actually are related to the thermal insulation thickness. Obviously, the increasing of thermal insulation thickness will increase the investment cost, but reduce the electrical power consumption. The problem is how we can compromise between the investment of insulation cost and the running cost of electrical power consumption.

Recently, many authors have studied the effect of thermal insulation thickness on the building transmission heat load and power consumption. The degree-day method is one of the well-known and the simplest methods used in the Heating, Ventilating and Air-Conditioning industry to estimate heating and cooling energy requirements. The yearly heating and cooling degree-hours are given both in tabular form and as counter maps [1] [2] [3]. Unsteady heat transfer throughout a composite wall exposed to explicit solar radiation and sinusoidal ambient temperature with Laplace s-domain is presented [4]. The solution is transformed back to the time domain using a series formula to provide an efficient alternative to a purely numerical treatment. A systematic approach for optimization of thermal insulation thickness is developed based on the life cycle cost analysis. Energy saving and payback periods are possible for rock wool and polystyrene insulation depending on the type of walls structure [5].

The effect of external roof color surface with periodic heat flow through a homogeneous flat solid concrete slab by solving the heat conduction equation was studied analytically [6]. They concluded that the influence of exterior color on reducing cooling loads has a minor effect when the concrete thickness from 15 cm and more. An experimental correlation between thermal conductivity and insulation thickness of certain insulation materials was presented to estimate the optimum thermal insulation thickness just by knowing the thermal conductivity [7]. An analytical investigation based on Complex Finite Fourier Transform to estimate the yearly building heat loads for two types of insulation materials and two typical wall structures [8]. They concluded that the most profitable case is the sandwich wall made of stone and brick with expanded polystyrene. The optimum insulation thickness of 5.7 cm provides energy savings of 58% and payback period of 37 months. Technical and economic optimal thermal insulation thickness of an external wall made from bricks was presented [9]. The data obtained from the mathematical model are illustrated in graphs to show the optimum thickness and minimum payback period.

A coupled heat and moisture transfer model is used to estimate the insulation thickness, lifecycle saving, and payback period [10]. They concluded that the lifecycle total cost of exterior wall using Expanded Polystyrene, XPS is lower than that using Extruded polystyrene, EPS insulation. The optimum thickness of XPS is between 0.053 and 0.069 m, and the optimum thickness of EPS is between 0.081

and 0.105 m, and the payback period varies from 23 to 31 months. A numerical solution of transient heat transfer through multilayer walls subjected to the average outdoor temperature and solar radiation was conducted [11] [12]. They concluded that the wall orientation had a significant effect on the optimum insulation thickness and energy savings. An economic model based on life-cycle cost analysis was used [13] [14]. They found that the optimum thermal insulation thickness increased with increasing the heating and cooling energy requirements, the lifetime of the building, the inflation rate, energy costs and thermal conductivity of insulation. Also, the insulation thickness decreased with increasing the discount rate, the insulation material cost, the total wall resistance, the coefficient of performance and the solar radiation. Recent studies are focused on energy saving in air-conditioned buildings according to the walls materials, insulation thickness, total life-cycle cost and walls orientation [15] [16] [17] [18]. Unsteady heat transmission through a composite wall was considered as one-dimensional problem to improve the accuracy of the optimal insulation thickness by using real meteorological data [19]. In the previous studies, there are very limited studies investigate the effect of solar radiation on the thermal insulation thickness and energy saving through 24 hours a day during four seasons, spring, summer, autumn and winter.

The objective of the present study is to investigate the thermal insulation thickness and energy saving in air-conditioned buildings. The analysis was conducted using expanded polystyrene thermal insulation in two types of outer wall surfaces with and without subjected to solar radiation. The hourly building heat load and power consumption with Cooling Load Temperature Deference, *CLTD*, method are estimated at various thermal insulation thickness and outside atmospheric conditions. The annual cooling degree-day, *CDD*, with including the solar radiation on the outer walls is presented to obtain the optimal insulation thickness and payback period.

2. Methodology

2.1 Residential Building Model

The transmission heat load is calculated according to the building construction materials. The building model we used was residential buildings or hotels with size of 18 × 18 × 3 m as illustrated in [20] [21]. The building was constructed from a common construction material and a single layer glazing windows with wooden doors as shown in **Figure 1(a)**. The building outer wall models, A and B are shown in **Figure 1(b)**. The building and walls model A is constructed from common materials; hollow bricks, heavy concrete, single layer clear glazing windows with aluminum frame, medium textile curtain, and wooden doors. The building and walls model B is the same as model A of construction materials in addition to 0 - 12 cm thermal insulation thickness of expanded polystyrene with thermal conductivity of 0.039 W/(m·K) in outer walls, and ceiling. The glazing windows are double layers clear glass with air gab of 1 - 3 cm [21].

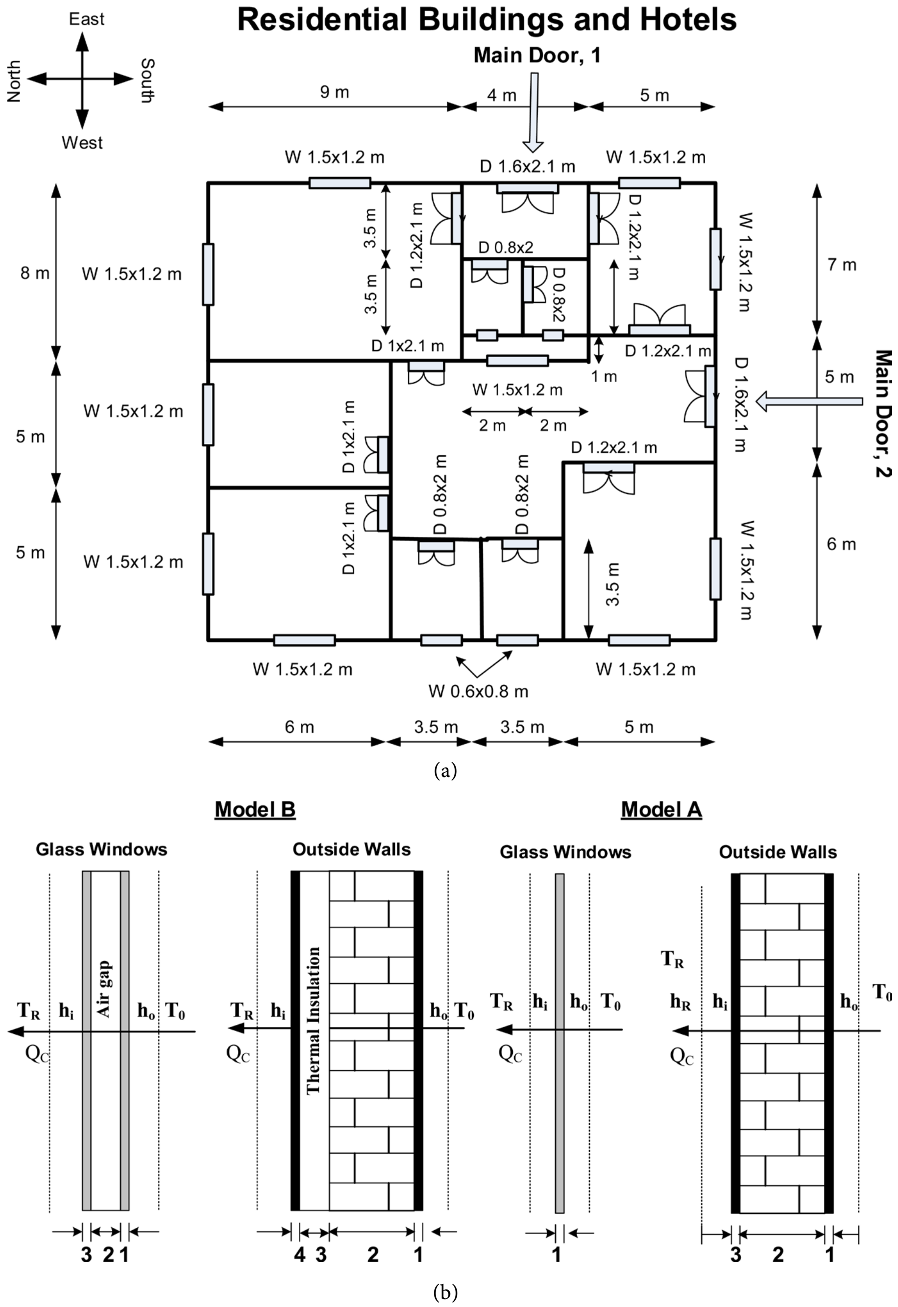

Figure 1. (a) Residential building layout; (b) Composite wall and glass window model A and B.

The building heat load is the amount of heat removed or added to the internal heat load to maintain the thermal comfort inside the building. The building heat load is two main parts. The first part is the sensible heat load which is a function of dry bulb temperature difference between outside and inside air condition, and radiation heat gain through glazing windows. The second part is the latent heat load which is a function of moisture transfer and vapor content inside the conditioned space. The building heat load in summer is the summation of internal and external heat loads. But, in winter is the difference between the internal and external heat loads. The building heat load was calculated using the Cooling Load Temperature Difference, *CLTD* method as illustrated in [22]. A computer program is prepared and tested under the constraints of the building orientation and north latitude and longitude. The data needed according to the *CLTD* method for outside air conditions and building construction materials are inserted as a subroutine to get the hourly building heat load at four seasons, spring, summer, autumn and winter.

2.2. Heat Transmission through Outer Surfaces Walls

The outer walls surfaces are subjected to instantaneous temperature $T_o(t)$ and solar radiation $I(t)$. The inner faces of walls come in contact with the indoor air which maintained at a fixed temperature of T_i, 24˚C to have a better thermal comfort. The one dimensional transient heat conduction model with constant thermal conductivity, density and heat capacity is used for this problem as follows:

$$\frac{\partial T(x,t)}{\partial t} = \frac{k}{\rho C_p} \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

where k is the thermal conductivity, ρ is the density and C_p is the specific heat of the wall material. To solve this problem, two boundary conditions and one initial condition are required. On both sides of the wall, convection boundary conditions are present. On the inner surface, at constant inside convection heat transfer coefficient is:

$$-k \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=L} = h_i \left(T_{x=L}(t) - T_i \right) \tag{2}$$

Whereas on the outdoor surface of the wall, the boundary condition at constant outside convection heat transfer coefficient can be written as:

$$-k \left. \frac{\partial T(x,t)}{\partial x} \right|_{x=0} = h_o \left(T_o(t) - T_{x=0}(t) \right) \tag{3}$$

where, h_o, h_i are the film convection heat transfer coefficient of outside and inside walls of building, and their values are $h_o = 22\ \text{W/m}^2 \cdot \text{K}$ and $h_i = 9\ \text{W/m}^2 \cdot \text{K}$ [23] [24]. T_i is the indoor air temperature; $T_o(t)$ is the outdoor air temperature; $T_{x=0}(t)$ is the wall inner surface temperature, and $T_{x=L}(t)$ is the wall outer surface temperature, respectively.

The steady-state solution of the problem at constant inside room temperature, constant inside and outside film convection heat transfer coefficient is taken. The function of outdoor temperature which is assumed to show sinusoidal variations during a 24-hour period, the instantaneous transmission heat transfer rate through walls from outside to inside is defined as,

$$Q_{trans}(t) = AU\left(T_o(t) - T_i \right) \tag{4}$$

where, $T_o(t)$ is the instantaneous outside temperature, and T_i is the indoor temperature. In steady state condition, the temperature difference is replaced by ΔT as mention in [20] according to Cooling Load Temperature Deference, $CLTD$ method and which is defined as,

$$\Delta T = (CLTD + LM)\cdot K + (25.5 - T_i) + (T_o(h) - 29.4) \tag{5}$$

where, $CLTD$ is the correction temperature difference according to latitude and surface orientation, LM is the correction factor of latitude, K is the color factor, $T_o(h)$ is hourly outside temperature, A is the outer walls area, and U is the overall heat transfer coefficient which is defined as,

$$\frac{1}{U} = \frac{1}{h_i} + \sum \frac{\Delta x}{k}\bigg|_{\text{materials}} + \frac{\Delta x}{k}\bigg|_{\text{insulation}} + \frac{1}{h_o} \tag{6}$$

To investigate the effect of thermal insulation thickness on the building heat load, we assumed all parameters are constant except thermal insulation thickness. The thermal resistance of composite wall can be written in a linear form as,

$$R = \frac{1}{U} = C_1 + C_2 \Delta x\big|_{\text{insulation}} \tag{7}$$

where C_1 and C_2 are constants, and the hourly heat transfer through the walls can be defined as,

$$Q_{trans} = UA\Delta T \tag{8}$$

In the numerical calculation, some assumptions were made. The thickness of the composite wall is small compared to other dimensions. So one-dimensional temperature variation is assumed. The layers are in good contact; hence the interfacial resistance is negligible. There is no heat generation. The variation of the wall materials thermal properties is negligible. The convection heat transfer coefficient and room temperature is constant.

2.3. Hourly outside Air Temperature and Relative Humidity

The hourly outside air conditions of dry bulb temperature and relative humidity are used in this analysis as illustrated in [20] [21]. The average data of dry bulb temperature, and relative humidity at latitude of 21.42˚N and longitude of 39.83˚E, Makkah City, Saudi Arabia, for each season are illustrated in **Figure 2** and **Figure 3**. The outside dry bulb temperature is continuously increased from sunrise to maximum value at 3:00 PM and decreased again to minimum value at night as shown in **Figure 2**. Opposite trend of dry bulb temperature, the relative humidity, which decreased from sunrise to minimum value at 3:00 PM and increased again to maximum value at night as shown in **Figure 3**.

3. Building Heat Loads

The total building heat loads in four season, summer, spring, autumn, and winter with constant internal heat loads are calculated at various thermal insulation thickness, Δx of 0 - 12 cm. The results of total building heat loads in four season are shown in **Figures 4-7**. We can see a clear and systematic effect of thermal

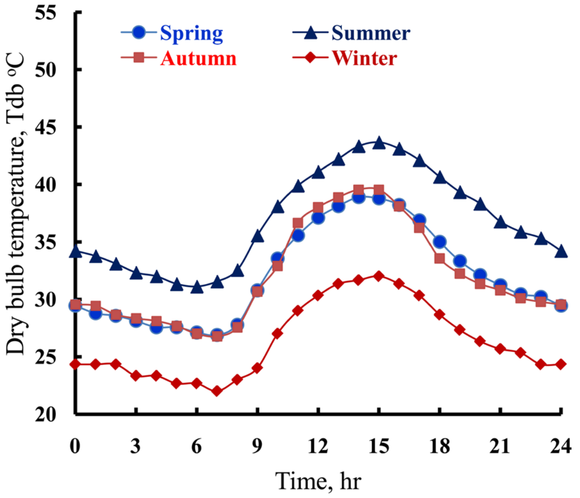

Figure 2. Dry bulb temperature at lat. 21.42˚N and long. 39.83˚E [21].

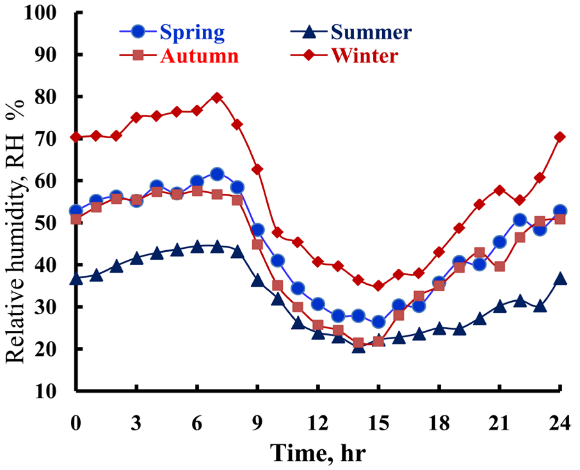

Figure 3. Relative humidity at lat. 21.42˚N and long. 39.83˚E [21].

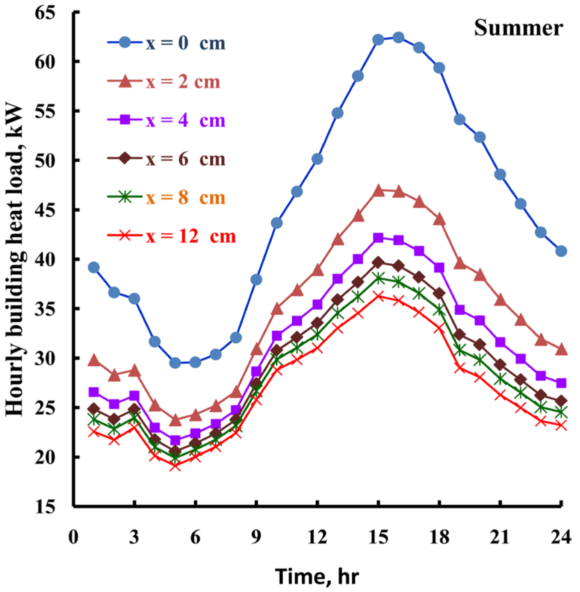

Figure 4. Building heat load in summer.

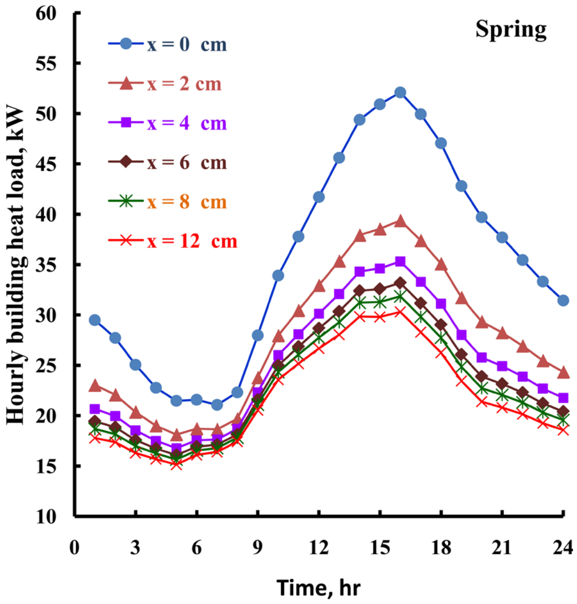

Figure 5. Building heat load in spring.

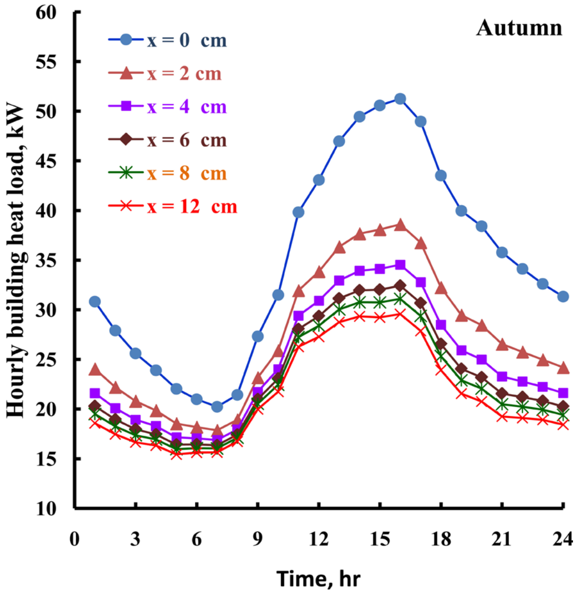

Figure 6. Building heat load in autumn.

insulation thickness, Δx, on the building heat load. Obviously, the building heat loads decrease with increasing thermal insulation thickness until 6 - 8 cm. Also, the trends of hourly building heat loads in four season are similar to the trends of outside air temperature as shown in Figure 2 and the maximum heat loads occurred at 3:00 PM. Otherwise, when thermal insulation thickness increased to 8 cm and above, the building heat loads did not decrease by considerable value.

4. Energy Consumption

A compression refrigeration machine with air cooled condenser is considered. A

Simple refrigeration cycle with DX evaporator of R134a was assumed at constant evaporating temperature of 8˚C and condensing temperature of 10 degrees above outside air temperature. The assumed isentropic compression efficiency is 70% and pressure drop of 0.5 bar in suction and delivery lines. The un-useful superheating is 1˚C in suction line and sub-cooled is 5˚C in liquid line. Cool Packprogram [25] is used in the analysis of refrigeration cycle to estimate the coefficient of performance, *COP*, and compressor consumed power. The hourly coefficients of performance through four season are illustrated in **Figure 8**.

It is clear that from **Figure 8**, the hourly coefficient of performance in summer is low than other season, because the outside weather is very hot, but in winter is larger than other season. The hourly compressor consumed power of the refrigeration machine is calculated at various outside air temperature and thermal insulation thickness, Δx , of 0 - 12 cm and the results are shown in

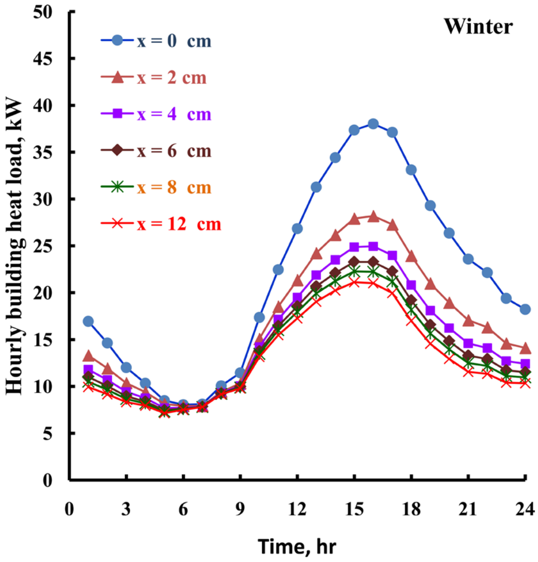

Figure 7. Building heat load in winter.

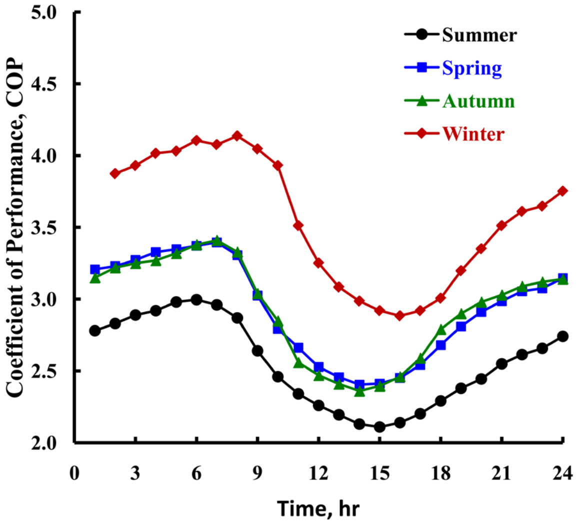

Figure 8. Coefficient of performance for refrigeration cycle.

Figures 9-12. We can see a clear effect of thermal insulation thickness, Δx, on the compressor consumed power, and it decreased with increasing the thermal insulation thickness until 6 - 8 cm. Also, with increasing thermal insulation thickness up to 8 cm, the decrease in compressor power consumption is very small.

5. Energy Saving

The saving percentage in building heat load or compressor consumed power are calculated as the difference between the value without insulation and with insulation to the value without insulation as,

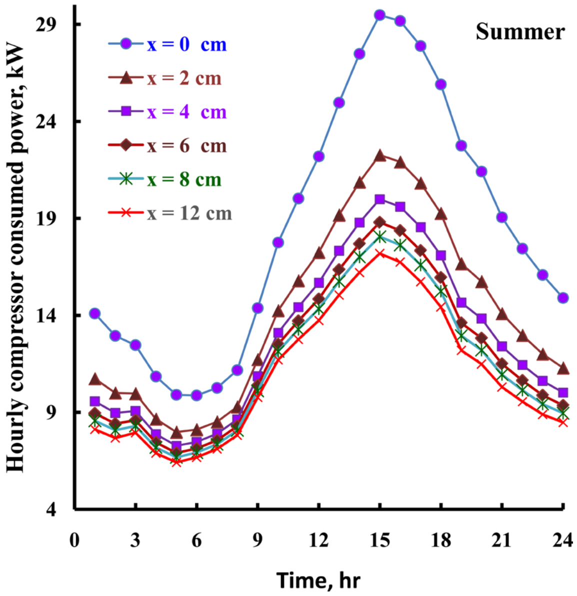

Figure 9. Compressor consumed power in summer.

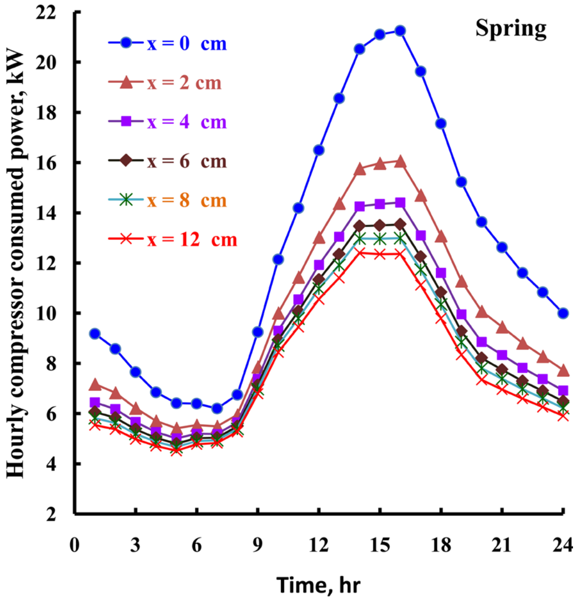

Figure 10. Compressor consumed power in spring.

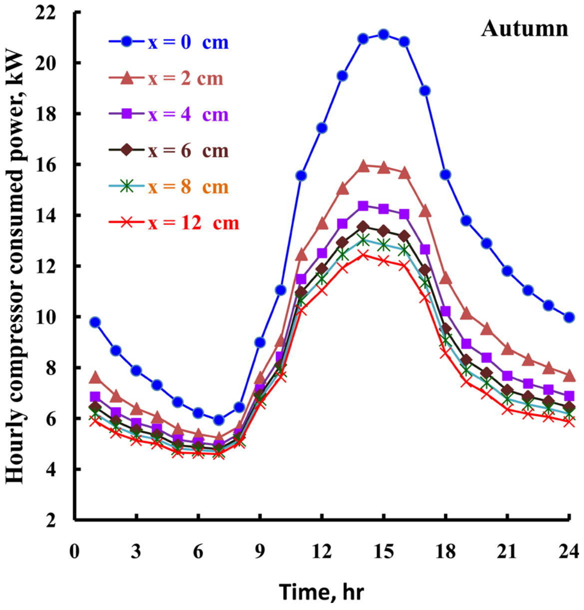

Figure 11. Compressor consumed power in autumn.

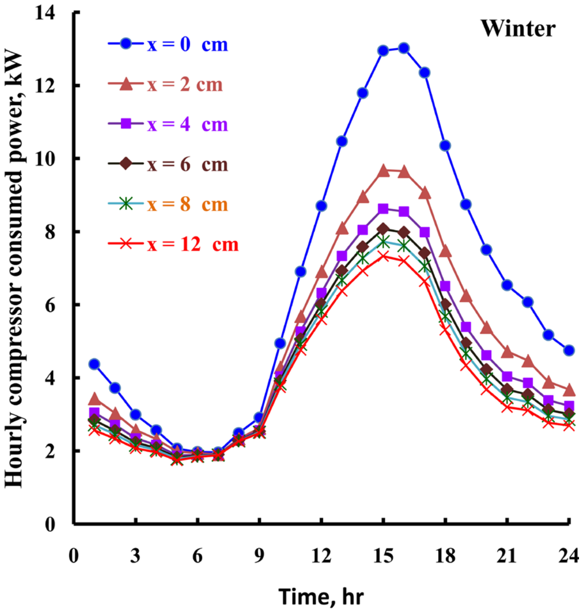

Figure 12. Compressor consumed power in winter.

$$\text{Energy Saving} = \frac{\text{Value without insulation} - \text{Value with insulation}}{\text{Value without insulation}} \quad (9)$$

Because the ratio between building heat load and the compressor consumed power is the coefficient of performance, the saving percentage of building heat load or compressor consumed power is the same value and trend. So, Figures 13-16 show the hourly saving percentage of compressor consumed powers in four seasons.

It is clear that from Figures 13-16, the hourly saving percentage increases from 9:00 AM to 8:00 PM of maximum value, and it decreases again. Also, the maximum saving percentage of consumed power is about 45% in summer, spring and autumn, but it attained to maximum value of 50% in winter.

Figure 17 shows the average daily saving percentage of compressor consumed power as a function of thermal insulation thickness in four season. The average daily saving increases with increasing the thermal insulation thickness until 6 - 10 cm. A small increase in the daily saving is found above 10 cm as shown in Figure 17. Figure 18 shows the average saving in compressor consumed power, and the average saving percentage in summer, spring, autumn and winter is 35.5%, 32.8%, 33.2% and 30.7% respectively. Also, the average saving percentage is about of 33% for all a year is illustrated in Figure 18.

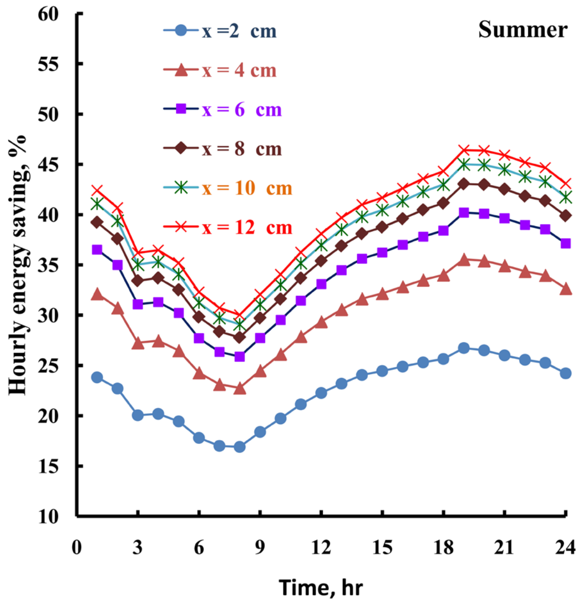

Figure 13. Hourly energy saving in summer.

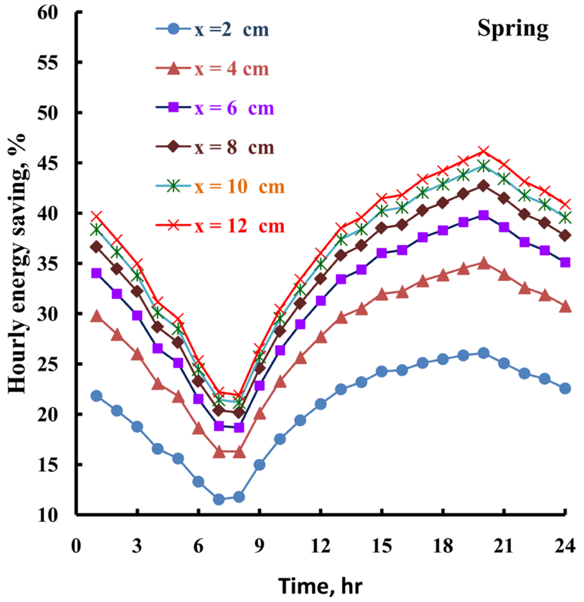

Figure 14. Hourly energy saving in spring.

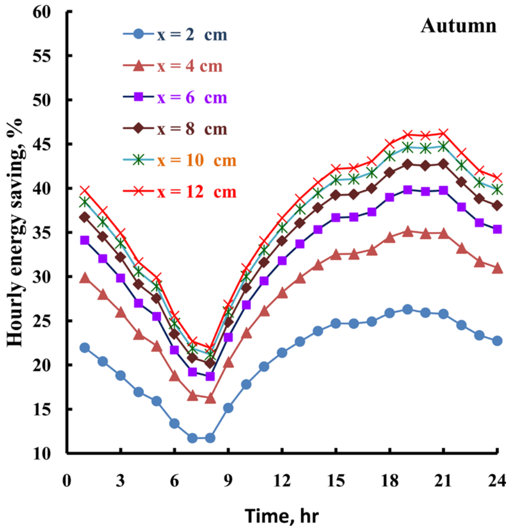

Figure 15. Hourly energy saving in autumn.

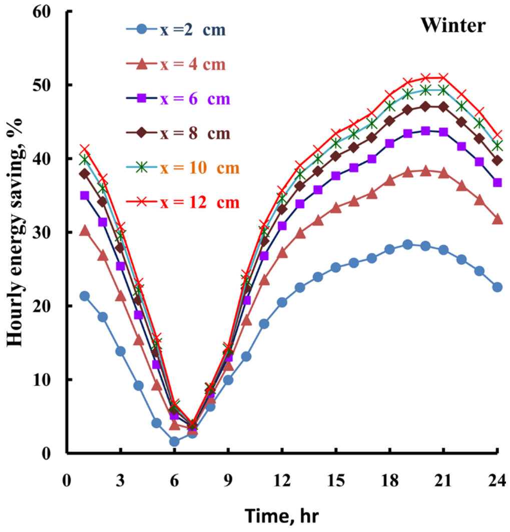

Figure 16. Hourly energy saving in winter.

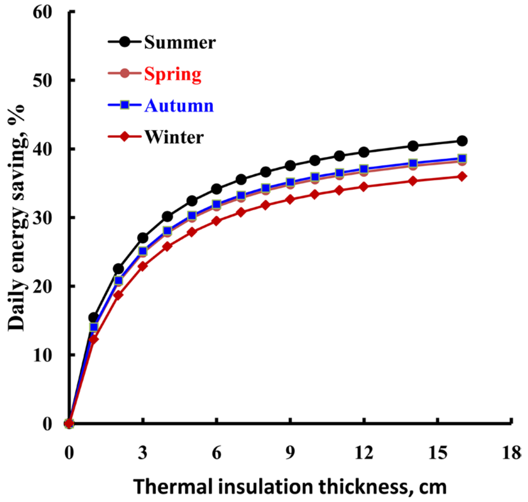

Figure 17. Average daily energy saving.

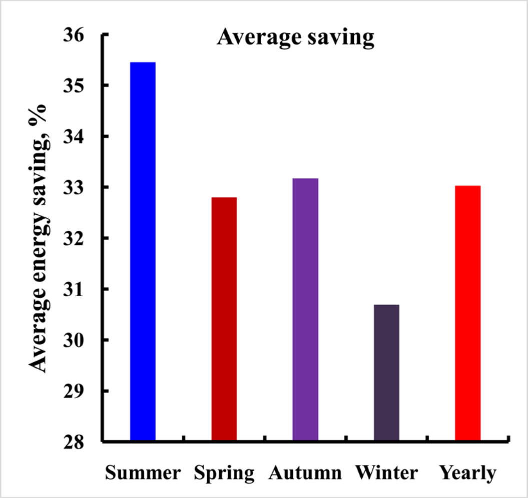

Figure 18. Seasonal and yearly average energy saving.

6. Economic Analysis

6.1. Optimum Insulation Thickness and Payback Period

It is important to determine the insulation thickness that minimizes the total cost, C_t, which is the cost of energy consumed plus the insulation cost [10] [11] as,

$$C_t = C_{enr} PWF + C_{ins} \Delta x_{ins} \tag{10}$$

where, $C_{enr}, \$/\left(\mathrm{m}^2 \cdot \mathrm{year}\right)$ is the yearly cost of the electric energy consumed relative to the thermal gains through one square meter of wall. *PWF*, is the present worth factor. $C_{enr}, \$/\left(\mathrm{m}^2 \cdot \mathrm{year}\right)$ is the cost of one cubic meter of insulation, and $\Delta x_{ins}, (\mathrm{m})$ is the thermal insulation thickness. Energy cost, C_{enr}, is depending on the yearly thermal gains through the outer surface walls, $q_T, \left(\mathrm{W}/\mathrm{m}^2\right)$. Electrical cost, $C_{el}, (\$/\mathrm{kWh})$ is the price of consumed energy and *COP* is the coefficient of performance of the refrigeration cycle for air-conditioning system as,

$$C_{enr} = \frac{q_T C_{el}}{COP} \tag{11}$$

The yearly transmission load per unit of wall area is estimated in (J/m^2) by the following equation,

$$q_T = 24/1000 U\, CDD = 0.024 U\, CDD \tag{12}$$

where, U as Equation (6) and *CDD* is the annual cooling degree-days which is defined as,

$$CDD = \sum_{1}^{365} \left(T_o - T_b\right) \tag{13}$$

where, T_o is the outside temperature and T_b is the base temperature which is equal to the inside conditioned building temperature which is 24˚C. The data of the annual cooling degree-days, *CDD*, are picked up from the National Climatic Data Center, meteorological stations of weather data of Egypt [27]. The effect of solar radiation and the hourly change of outside temperature are included in Equation (13) from Equation (5), and the annual cooling degree-days are modified as,

$$CDD_{mod}=\sum_{1}^{365}\left(\left(CLTD+LM\right)K-3.9+\left(T_o-T_b\right)\right) \tag{14}$$

We used the absolute difference of $\left(T_o-T_b\right)$ to treat the cooling case in summer and heating case in winter. The annual energy cost for unit wall surface area can be rewritten as,

$$C_{enr}=\frac{0.024CDD_{mod}C_{el}}{\left(R_{wt}+\left(\Delta x/k\right)_{ins}\right)COP} \tag{15}$$

where, R_{wt} is the total composite wall thermal resistance except insulation thermal resistance. The lifecycle total cost, C_t is the energy cost and insulation cost can be rewrite as,

$$C_t=\frac{0.024CDD_{mod}}{COP}\frac{1}{R_{wt}+\left(\Delta x/k\right)_{ins}}C_{el}PWF+C_{ins}\Delta x_{ins} \tag{16}$$

The optimal insulation thickness, Δx_{opt}, is the thickness of the insulation layer that corresponds to that minimizing the total cost as,

$$\Delta x_{opt}=\left(0.024\frac{CDD_{mod}k_{ins}C_{el}PWF}{C_{ins}COP}\right)^{0.5}-k_{ins}R_{wt} \tag{17}$$

The lifecycle saving, LCS, is the difference between the saved energy cost over the lifetime and the insulation cost [10] as,

$$LCS=PWF\,\Delta E_C-C_{ins}\Delta x_{ins} \tag{18}$$

where, ΔE_C the saved energy which is the difference between the annual energy consumption per unit area of the wall without and with insulation under cooling condition as,

$$\Delta E_C=\frac{0.024CDD_{mod}C_{el}}{COP}\left(\frac{1}{R_{wt}}-\frac{1}{R_{wt}+\left(\Delta x/k\right)_{ins}}\right) \tag{19}$$

And the present worth factor PWF is defined as,

$$PWF=\begin{cases}\dfrac{1+i}{d-i}\left[1-\left(\dfrac{1+i}{1+d}\right)^{n}\right] & \text{at } i\neq d\\[2ex] \dfrac{n}{1+i} & \text{at } i=d\end{cases} \tag{20}$$

The payback period N_P (years) can be obtained by setting LCS to be zero as,

$$N_P=\begin{cases}\dfrac{\ln\left[1-\dfrac{C_{ins}\Delta x_{ins}}{\Delta E_C}\dfrac{d-i}{1+i}\right]}{\ln\left(\dfrac{1+i}{1+d}\right)} & \text{at } i\neq d\\[2ex] \dfrac{C_{ins}\Delta x_{inc}\left(1+i\right)}{\Delta E_C} & \text{at } i=d\end{cases} \tag{21}$$

where, n is the yearly lifecycle, i is the currency inflation rate and d is the interest rate. The parameters used in the calculation of the total cost, the optimum insulating thickness and payback period are given in Table 1.

Two main parameters are affecting the optimal thermal insulation thickness, the running cost of electrical energy and the investment cost of the thermal insulation volume. The tariff of electrical energy of 0.087 \$/kWh and expanded polystyrene thermal insulation cost of 53 $\$/\mathrm{m}^3$ are used in this analysis. The optimal thickness of thermal insulation and payback period is calculated in two cases. The first case is neglecting the effect of solar radiation on the outer walls surfaces and ceiling, and he cooling degree-days *CDD* as Equation (13). The second case is taking into account the effect of solar radiation. The modified cooling degree-days, CDD_{mod}, as Equation (14) is used only through the day time from sunrise to sunset. But from sunset to sunrise, the *CDD* is calculated by Equation (13) as the absolute value between outside and inside temperature of $\left(T_o - T_b\right)$ to treat the cooling and heating cases.

6.2. Solar Radiation Effect on Total Cost

Without exposed the outer walls surfaces to direct solar radiation, Figure 19 shows

Table 1. Parameters used in calculation.

Parameters	Present data	Reference [26]
C_{el} (\$/kWh)	0.087	0.1102
C_{ins} ($\$/\mathrm{m}^3$)	53	53
COP	3.0	-
R_{wt} ($\mathrm{m}^2 \cdot \mathrm{K/W}$)	0.5139	0.774
Inflation rate, *i*	1%	8.53%
Interest rate, *d*	5%	9%
Cycle life time, *n*	10 years	15
PWF	8.127	8.58

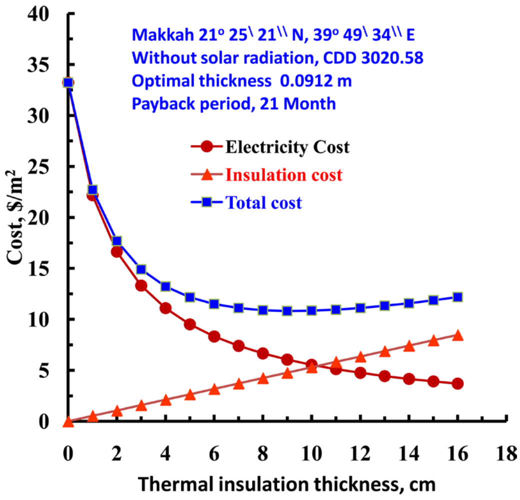

Figure 19. Optimal thermal insulation thickness, without solar radiation.

the energy cost, thermal insulation cost and total cost as a function of insulation thickness. The running cost of electrical energy is decreased with increasing the thermal insulation thickness, but the cost of thermal insulation increases with increasing the insulation thickness. The thermal insulation thickness is increased gradually from 0 to 16 cm. The total cost is gradually decreasing to a minimum value and increased again. At this point the optimum value of insulation thickness is appointed and it is obtained of 9.12 cm and payback period is 19 months. According to the previous studies [10] [11], similar results are illustrated and the optimal thickness of extruded polystyrene (XPS) was between 5.3 and 6.9 cm and expanded polystyrene (EPS) was between 8.1 and 10.5 cm. The payback period varies from 22 to 30 months.

With including the effect of solar radiation on the outer walls surfaces, the modified annual cooling degree-days, CDD_{mod}, was calculated from Equation (14). The values of correction factor, LM, and the Cooling Load Temperature Difference, $CLTD$, are illustrated in **Table 2**. The values of LM in a certain month at

Table 2. Latitude correction factor, LM [22].

Latitude	Months	North	East/West	South	Horizontal
16°	December	−2.2	−2.2	7.2	−5.0
	January/November	−2.2	−2.2	6.6	−3.8
	February/October	−1.6	−1.1	3.8	−2.2
	March/September	−1.6	−0.5	0.0	−0.5
	April/August	−0.5	−0.5	−3.2	0.0
	May/July	2.2	−0.5	−3.8	0.0
	June	3.3	−0.5	−3.8	0.0
24°	December	−2.7	−3.8	7.2	−7.2
	January/November	−2.2	−3.3	7.2	−6.1
	February/October	−2.2	−1.6	5.5	−3.8
	March/September	−1.6	−0.5	2.2	−1.6
	April/August	−1.1	−0.5	−1.6	0.0
	May/July	0.5	0.0	−3.3	0.5
	June	1.6	0.0	−3.2	0.5
32°	December	−2.7	−4.4	6.6	−9.4
	January/November	−2.7	−4.4	6.6	−8.3
	February/October	−2.2	−2.2	6.1	−5.5
	March/September	−1.6	−1.1	3.8	−2.7
	April/August	−1.1	0.0	0.5	−0.5
	May/July	0.5	0.0	−1.6	5.0
	June	0.5	0.0	−2.6	1.1

The average value of $CLTD$ for ceiling 16.38°C, for walls, North 6.54°C, East/West 12°C, South 9.33°C. The middle color factor, $k = 0.65$.

wall orientation are calculated by interpolation at a given latitude.

Figure 20 shows the effect of solar radiation on energy consumption, insulation thickness and total cost of walls and ceiling. The thermal insulation thickness is illustrated from 0 - 16 cm and the optimal insulation thickness for walls is obtained of 10.06 cm and for ceiling is 11.68 cm. The payback period is found of 18 - 20 months. When we used the average value of CDD_{mod} for walls and ceiling, the optimal insulation thickness is obtained of 10.89 cm and payback periods of 19 months. We observed that the walls optimum thermal insulation thickness with the effect of solar radiation is increased by 10.31% than walls without solar radiation effect.

It is important to calculate the optimal thermal insulation thickness and payback periods with the effect of solar radiation for some cities in Egypt. So, we used the same procedures for some important cities in Egypt from latitude of 24˚N to 32˚N and the results are illustrated in Table 3. Figure 21 shows the effect of annual degree-days on the optimum insulation thicknesses and payback period for the present data. Obviously, the present data for both walls and ceiling shows the optimum thermal insulation thickness is increased from 7.12 -

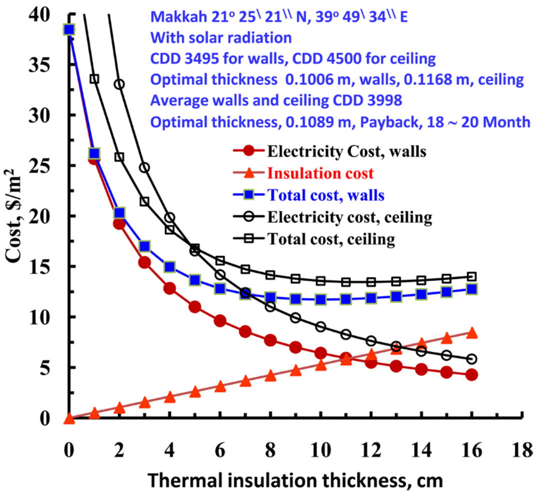

Figure 20. Optimal thermal insulation thickness, with solar radiation.

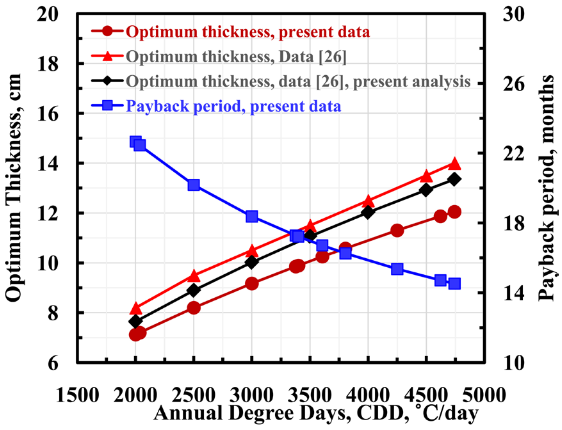

Figure 21. Effect of annual degree days on optimum insulation thickness.

12.05 cm with increasing the annual degree-days. Because of the roof is treated as a horizontal slab without suspended ceiling and the *CLTD* larger than walls. So, the optimum insulation thickness for ceiling is larger than walls by about 20.19%. Also, the payback period is decreased with increasing the optimum insulation thickness as shown in **Figure 21** and it decreased from 23 months to 15 months.

The optimum insulation thickness of data which sited in reference [26] is calculated by our analysis with parameters from **Table 1** and compared with sited data. The estimated result of optimum insulation thickness by our analysis is a little lower than the sited data by about 4.9%. Also, it is observed that the sited data in reference [26] is larger than our data by about of 12.76% at the same degree-days, maybe due to the different values of electrical energy cost.

6.3. Effect of Insulation Materials on Optimum Thickness

To investigate the effect of insulation materials on the optimum thickness periods, five types of thermal insulation materials are used and its properties are illustrated in **Table 4**. **Figure 22** shows the optimum insulation thickness for various insulation materials with annual degree-days. The optimum insulation thickness increases with increasing thermal conductivity of the insulation materials as Extruded polystyrene (XPS), Expanded polystyrene (EPS), and Glass wool (GW). Also, the optimum insulation thickness depends on the price of the

Table 3. Optimum insulation thickness for some cities in Egypt.

City	Location Latit./Longit.	Object	CDD_{mod} Degree-days	Δx_{opt} (cm)	Payback years
Alexandria	31°12'20"N	Walls	2000	7.12	1.888
	29°55'28"E	Ceiling	3378	9.85	1.44
Cairo	30°01'59"N	Walls	2035	7.2	1.871
	31°14'00"E	Ceiling	3400	9.89	1.435
Assiut	27°11'00"N	Walls	3805	10.58	1.355
	31°10'00"E	Ceiling	4743	12.05	1.210
Aswan	24°05'20"N	Walls	3606	10.25	1.393
	32°53'59"E	Ceiling	4622	11.87	1.226

Table 4. Properties of insulation materials [26].

Insulation materials	*k* (W/m·K)	Cost ($/m^3)
Extruded polystyrene (XPS)	0.031	79
Expanded polystyrene (EPS)	0.039	53
Glass wool (GW)	0.04	27
Polyurethane (PUR)	0.024	118
Polyisocyanurate (PIR)	0.023	105

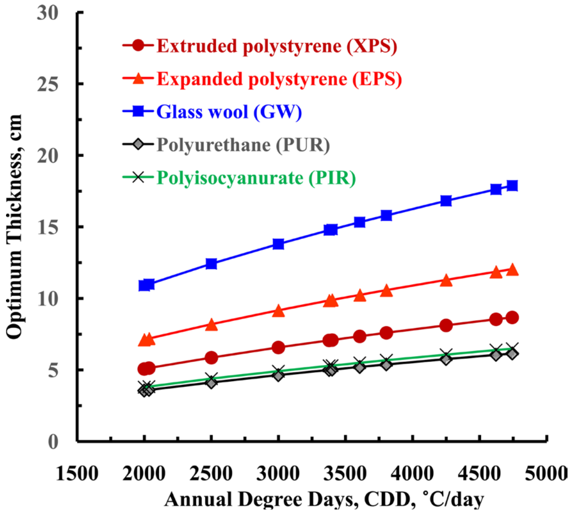

Figure 22. Effect of insulation materials on optimum insulation thickness.

insulation material, and it decreases as the price of the insulation materials increased at the same thermal conductivity, Polyurethane (PUR) and Polyisocyanurate (PIR).

7. Conclusion

The building heat load and energy consumption by air conditioning system are proportional to the environmental conditions and solar radiation intensity. The building heat load was estimated for two models of walls. The walls models are constructed from common materials with 0 - 16 cm of thermal insulation thickness in the outer walls and ceilings, and double-layers glazing windows. The optimal thermal insulation thickness and payback period are estimated for residential building from latitude of 16˚N to 32˚N. The average saving percentages in energy consumption in summer, spring, autumn and winter are estimated as, 35.5%, 32.8%, 33.2% and 30.7% respectively. But the yearly average saving percentage is about of 33.5%. Economic analysis is conducted with the annual cooling degree-days and life cycle total cost to estimate the optimum thickness and payback periods. The optimal thermal insulation thickness for some Egyptian cities is found between 7 - 12 cm for various annual cooling degree-days of 2000 to 5000 and five types of insulation materials. Without including the effect of solar radiation, the optimum insulation thickness of ceiling is larger than walls by about 20%. But with including the effect of solar radiation, the optimal thermal insulation thickness for walls is about 10 cm and ceiling of 11.68 cm, and payback period is found between 15 - 24 months.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Nomenclature

A: wall surface area (m^2)
C_{el}: energy cost (\$/kWh)
C_{ins}: insulation cost (\$/m^3)
CDD: cooling degree-day (˚C/day)
$CLTD$: cooling load temperature difference (K)
COP: coefficient of performance (–)
h: heat transfer coefficient ($\mathrm{W \cdot m^{-2} \cdot K^{-1}}$)
K: color factor (–)
k: thermal conductivity ($\mathrm{W \cdot m^{-1} \cdot K^{-1}}$)
LM: latitude correction factor (–)
Q: heat transfer rate (W)
R: wall thermal resistance ($\mathrm{m^2 \cdot K \cdot W^{-1}}$)
T: temperature (K)
t: time (s)
x: thickness (m)
U: overall heat transfer coefficient ($\mathrm{W \cdot m^{-2} \cdot K^{-1}}$)

Subscript

enr: energy
el: electrical
o: outside
opt: optimum
i: inside
ins: insulation
$trans$: transmission
wt: wall materials