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Exchange rate policy in the pacific: an evaluation of currency basket regimes

Naoyuki Yoshino, Matthias Helble and Ahmad Danu Prasetyo*

The Pacific island countries have opted for exchange rate regimes with varying degrees of flexibility. Whereas several microstates have adopted an external currency as their legal tender, others have decided to use a basket currency, and yet others have chosen a managed float. The choice of exchange rate regime can have far reaching economic consequences. In the paper, we study the basket currency arrangements by Fiji, Samoa, the Solomon Islands, and Tonga. We first build a new four-country exchange rate model that illustrates how monetary authorities should best determine the weights of the basket currencies given certain macroeconomic objective functions. In this model, we explicitly include tourism flows. In the second part of the paper, we estimate the *de facto* weights of foreign currencies in the currency basket of the four countries. We show how the composition has changed amid the global financial crisis. Finally, we demonstrate that the current weights are not optimal compared with the predictions of our model.

Introduction

The developing countries in the Pacific region are a heterogeneous group of economies.[1] Most markedly, their economic development differs strongly; the gross domestic product (GDP) per capita ranges from about $1000 in Timor-Leste to more than $20 000 in the Cook Islands.[2] Similarly, the economic size of the countries differs sharply, with a GDP of $38 million in Tuvalu compared with $15.5 billion in Papua New Guinea (PNG). However, when it comes to optimal exchange rates, the Pacific island countries (PICs) are faced with similar challenges: First, all of the Pacific economies are small, and many can be defined as microstates, as they have less than 200 000 inhabitants.[3] Second, most of them fall into the category of lower-middle-income countries by the definition of the World Bank.[4] These factors make it difficult to allocate the necessary financial and technical resources to establish and run a central monetary authority. A third similarity among Pacific economies that limits their exchange rate choice is that they all have very underdeveloped

* Naoyuki Yoshino, Asian Development Bank Institute (ADBI), Tokyo, Japan. Matthias Helble, Research Economist, ADBI, Tokyo, Japan. Email: mhelble@adbi.org. Ahmad Danu Prasetyo, Lecturer, Bandung Institute of Technology, Indonesia. Email: ahmad.danu@sbm-itb.ac.

1 In this paper, we focus on 4 of the 14 Pacific developing countries that are member of the Asian Development Bank, namely Cook Islands, Federated States of Micronesia, Fiji, Kiribati, Marshall Islands, Nauru, Palau, Papua New Guinea, Samoa, Solomon Islands, Timor-Leste, Tonga, and Tuvalu.
2 Unless otherwise indicated, currencies are in US dollars.
3 Countries with fewer than 200 000 inhabitants are typically referred to as microstates (IMF 2013a).
4 For 2013, the World Bank defined lower-middle-income economies as those with a Gross National Income (GNI) per capita, calculated using the World Bank Atlas method, of more than $1036 but less than $4085.

financial markets. Fourth, exchange rate markets in the Pacific are typically shallow and tend to be highly volatile. All these elements make it difficult for countries to choose their optimal exchange rate regime.

Table 1 lists all Pacific developing member countries (DMCs) of the Asian Development Bank and their exchange rate according to the International Monetary Fund (IMF) *de facto* classification (IMF 2013b). Given the constraints explained previously, it is not surprising that most Pacific DMCs have exchange regimes that are characterised by a very low degree of flexibility. Eight out of 14 Pacific DMCs uses the currency of another country as sole legal tender and have thus given up their monetary control. All these Pacific DMCs have a population of less than 150 000, except for Timor-Leste, with 1.3 million.

Several of the larger Pacific economies maintain exchange rate regimes that offer more flexibility. Four, namely Fiji, Samoa, the Solomon Islands, and Tonga, have adopted a currency-basket regime. In a currency-basket regime, the weights in the basket typically correspond to the importance of the respective trading or financial partners. Fiji, for example, uses trade weights to determine the shares of the basket currencies. Finally, two Pacific economies have an even more flexible regime in place. PNG and Vanuatu apply an exchange rate regime described as 'managed floating', in which the central banks manipulate the exchange rate without having a specific exchange rate path or target.

The objective of this paper is to study the exchange rate choices of the four economies that use a currency-basket regime (Fiji, Samoa, the Solomon Islands, and Tonga). We chose to focus on currency-basket regimes because a currency-basket regime offers some flexibility, as weights in the basket can be adjusted. We build a new theoretical four-country model that allows determining the optimal weights in the basket given a certain objective function by the government. We are thus able to predict the optimal weights and contrast them with the actual weights applied by the countries.

The paper is structured as follows: after the brief literature review, The Optimal Basket Weights section presents a simple four-country basket currency model. The Simulation of the Theoretically Optimal Shares section presents a simulation of optimal shares assuming a specific loss function by the government. Finally, applying the method introduced by Frankel and Wei (1994), in the Estimating *De Facto* Basket Weights section, we determine the *de facto* weights in the currency basket of the four selected countries and show how it has evolved over time.

Table 1
Exchange rate arrangements in pacific developing member countries

Pacific DMC	Name of currency	*De facto* classification (IMF)	Population	GDP per capita ($)
Cook Islands	New Zealand dollar	External currency	15 225	21 490
Fed. States of Micronesia	US dollar	External currency	102 908	3057
Fiji	Fiji dollar	Currency basket	863 073	4652
Kiribati	Australian dollar	External currency	111 117	1528
Marshall Islands	US dollar	External currency	54 550	3333
Nauru	Australian dollar	External currency	10 660	7502
Palau	US dollar	External currency	17 862	13 761
Papua New Guinea	Kina	Managed floating	7 570 686	2043
Samoa	Tala	Currency basket	187 372	3641
Solomon Islands	Solomon Islands dollar	Currency basket	626 247	1858
Timor-Leste	US dollar	External currency	1 306 000	1063
Tonga	Pa'anga	Currency basket	103 347	4619
Tuvalu	Australian dollar	External currency	11 099	3434
Vanuatu	Vatu	Managed floating	271 089	2951

Note: Population and economic figures are from 2013.
Source: International Monetary Fund (IMF) (2013b); Asian Development Outlook Database (2014).
DMC = developing member country; GDP = gross domestic product; IMF = International Monetary Fund.

Literature review

The literature on optimal exchange rate policy is vast and has been constantly expanding and evolving. As a consequence, recommendations by leading international organisations on optimal exchange rate policy choices for developing countries have also been changing. In the 1990s, it became popular for developing and emerging economies to peg the exchange rate against leading international currencies. However, the Asian financial crisis of 1997 showed the risk of such a policy. Sharp reversals of capital inflows triggered the collapse of several Asian currencies and led to sharp economic contraction. After the Asian financial crisis, the IMF tended to recommend either fully floating regimes or hard pegs. This became known as bipolar prescription (Ghosh and Ostry 2009). Intermediate regimes, such as crawling arrangements or managed floats, were not advised because leading economists, such as Obstfeld and Rogoff (1995), argued that they made countries more vulnerable to crisis.

Following the bipolar prescription, many countries did indeed abandon intermediate regimes and adopted more flexible regimes. However, this trend came to an end in the mid-2000s (Ghosh et al. 2014). Increasingly, emerging countries started to fear that a freely floating exchange rate could lead to strong currency volatility, which could be deleterious to economic growth. In addition, IMF research indicated that historically, intermediate exchange rate regimes exhibited the best growth performance (Ghosh and Ostry 2009). Their research into the global financial and economic crisis that began in 2007 also indicated that, for emerging countries in Europe, hard pegs triggered strong declines in economic output and harsh current account reversals. The adequacy of hard pegs to weather economic shocks was thus called into question again.

Today, the majority of countries have an intermediate exchange rate regime in place. Latest research by the IMF suggests that intermediate regimes are more vulnerable to crisis than free floats (Ghosh et al. 2014). However, given that central bankers in developing countries have a preference for at least some control over the exchange rate, Ghosh et al. (2014) recommend managed floats, as they offer almost the same advantages as pure floats.

While at the international level, there seems to be a preference for more intermediate regimes, for microstates, such as most of the PICs, the choice is less clear. According to Imam (2010), there are several reasons why microstates usually fare better with a fixed exchange rate regime. First, microstates typically lack the necessary institutional infrastructure to operate monetary policy, including the qualified professional staff necessary to manage a central bank. Second, as financial markets are typically underdeveloped in microstates, monetary policy will be largely driven by exchange rate considerations and therefore cannot be used proactively to influence economic activity. Third, the volatility of the exchange rate can be excessive because foreign exchange markets are illiquid. As a result, floating exchange rates in microstates are likely to become *de facto* fixed over time, with the authorities intervening to smooth fluctuations. Fourth, central banks of microstates typically lack credibility, resulting in a 'fear of floating' and high levels of dollarisation. A hard peg thus allows microstates to import credibility. And finally, microstates are often sufficiently well integrated with the former colonial power or regional partner that they almost naturally form an optimal currency area with them.

These reasons explain why the empirical literature on monetary policy choices in the PICs often recommends the adoption of an external currency. For example, Freitag (2011), in his review of the currency and trade experiences of the six Pacific states that issue their own currencies (Fiji, PNG, Samoa, the Solomon Islands, Tonga, and Vanuatu), finds that a large and increasing proportion of the trade, and thus the reserves, of these countries is denominated in US dollars. Using gravity model estimation, he suggests that these Pacific states should replace their own currencies with the US dollar as it would substantially stimulate the countries' trade and accelerate economic growth. In addition, dollarisation would reduce transaction costs with East Asia and for most trade in global resources. Furthermore, he argues that the loss of seigniorage would be outweighed by cost savings in operating central banks. Jayaraman and Narayan (2011) evoke similar advantages from studying fixed exchange rate regimes of small developing countries. They highlight the advantage of a fixed exchange rate in terms of facilitating capital mobility, promoting long-term investment, and lowering uncertainty.

However, as alluded to earlier, a pegged regime has advantages as well as disadvantages. As Ghosh and Ostry (2009) explain, pegged regimes severely constrain the use of other macroeconomic policies following the logic of the 'impossible trinity'. In addition, pegged regimes are associated with greater susceptibility to currency and financial crises (but countries with floating regimes are not entirely immune). Finally, pegged and intermediate exchange rate regimes impede timely external adjustments.

Despite the arguments against pegged regimes, all Pacific DMCs with a population of less than 150 000 (and Timor-Leste, with a much larger population) have decided to adopt an external currency (Table 1). For those eight countries, the perceived benefits of using an external currency outweigh the expected costs. Assuming their choice as given, one might ask whether the external currency is the optimal one and whether, given the increasing integration with Asia, a switch in the external currency will be necessary in the near future.

The second most prominent exchange rate regime found in the Pacific DMCs is currency baskets. The IMF defines a basket currency as a currency that bases its value on a portfolio of other currencies with different weights (Duttagupta et al. 2005). The basket is formed from the currencies of major trading or financial partners, and weights reflect the geographical distribution of trade, services, or capital flows. A basket currency is sometimes also called a soft-peg policy because the value of the currency is pegged to several major currencies instead of just one.

Yoshino et al. (2004) explain the advantages and disadvantages of having a basket currency policy. There are two positive aspects. First, the exchange rate fluctuations are typically lower than a hard-peg, because the exchange rate risk is more dispersed. Second, the weights in the basket may be used as an additional policy tool to achieve a given exchange rate objective. Authorities can minimise the deviation from their policy goal by choosing the values for these weights accordingly. This need for frequent, if not constant, readjustment can be positive or negative. It is positive if the composition of the basket moves in the direction targeted by the monetary authorities, because the monetary authorities do not then need to intervene in the market and can save the foreign reserves using minimal effort to stabilise the value of the currency. However, if the movement goes in the opposite direction, then the monetary authorities need to adjust the weights of the currencies in the basket, which can be costly in terms of foregoing foreign reserves. Finally, there is an important caveat. In order to realise the advantages of a currency basket, the country should denominate trade in the different currencies. Even if the effects on the exchange rates are more dispersed, countries will not gain from the basket if their use of currencies in trade is not correspondingly diverse.

For PICs with a currency basket, the question is whether the weights in the basket reflect the current trade patterns. According to economic theory, the external currency of choice should be the currency of the major trade and finance partner. For the PICs, financial links with the rest of the world are not well documented. This contrasts with documentation on trade flows. We have therefore chosen to study the flows of trade in goods and services to evaluate the appropriateness of the weights in the currency baskets.

Finally, two countries, PNG and Vanuatu, maintain a managed float. According to Ghosh et al. (2014), managed floats can be as safe as full floats if they guarantee almost full flexibility. This means that the countries not only have to apply a *de jure* managed float but also *de facto*; otherwise, there is an increased probability of a financial crisis. Similarly, Jayaraman and Narayan (2011)argue that more flexible regimes are only successful if fiscal policies are disciplined, and the complementary institutions enjoy independence and are transparent.

In summary, the literature on optimal exchange rate regimes has evolved over the past two decades and so have the exchange rate choices of countries around the world. Most of the smaller Pacific countries have opted for an external currency as legal tender. There are strong arguments in favour of this choice. The focus of this paper is on the countries that have adopted a currency basket. In the next section, we propose a simple theoretical model to calculate the optimal basket weights, given a loss function of the government that aims to minimise fluctuations of GDP and exchange rates.

The optimal basket weights

Yoshino et al. (2003) argue that managing the shares in the basket currency optimally can help governments to reduce GDP volatility. We build on their model and extend it in two novel ways: First, whereas Yoshino et al. (2003) develop a model including three countries, we add another fourth country in the model. Adding another country has the advantage of better reflecting the current situation of the four Pacific countries with a currency basket. Most of them include more than two foreign currencies in their baskets. The model allows us to determine the optimal weights for the basket currencies of the four PICs. Second, we explicitly model trade in services. Previous models only included trade in goods. However, we know that the services' trade can play a vital role in an economy, especially when tourism is one of the main industries.

The model

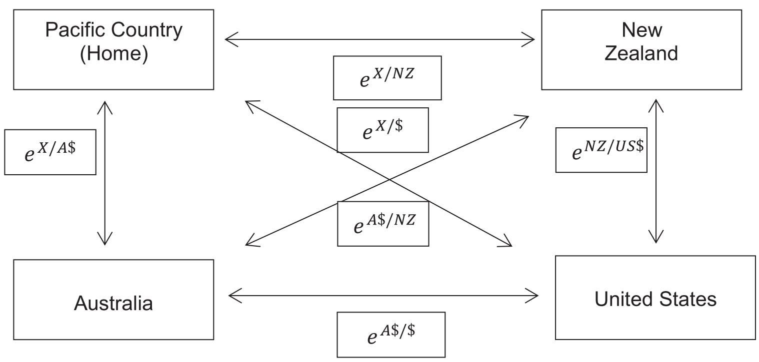
Let us assume that a country's currency basket contains three currencies, namely the US dollar, the Australian dollar, and the New Zealand dollar. The shares of these three currencies in the basket determine the value of the exchange rate of the domestic currency (X) towards all foreign currencies. Furthermore, we assume that there are four countries: the Pacific country, Australia, New Zealand, and the USA. The Pacific country is labelled 'Home'. We assume that domestic and foreign assets are imperfect substitutes, whereas US, Australian, and New Zealand assets are perfect substitutes for domestic investors (Figure 1).

Let $e^{X/US}$, $e^{X/AU}$, and $e^{AU/US}$ denote the exchange rate of the Pacific currency against the US dollar, the Pacific currency against the Australian dollar, and the Australian dollar against the US dollar. Because one of the three exchange rates is not independent, the Pacific currency–US dollar exchange rate can be expressed as

$$e^{X/US} = e^{X/AU} + e^{AU/US}. \tag{1}$$

Following the same logic, we know that the following relationships must hold:

Figure 1
Four-country exchange rate relationship chart



Source: Authors.

$$e^{X/US} = e^{X/NZ} + e^{NZ/US}, \quad (2)$$

$$e^{X/NZ} = e^{X/AU} + e^{AU/NZ}, \quad (3)$$

$$e^{X/AU} = e^{X/NZ} - e^{AU/NZ}. \quad (4)$$

We assume that the monetary authority would adjust money supply by intervening in the foreign exchange market in order to maintain the value of the basket at a constant level θ.[5] If v is the weight of the US dollar, w is the weight of the Australian dollar, and $(1-v-w)$ is the weight of the New Zealand dollar in the basket, then the value of the basket can be described as

$$ve^{X/US} + we^{X/AU} + (1-v-w)e^{X/NZ} = \theta. \quad (5)$$

Rearranging (5), we obtain

$$e^{X/AU} = \frac{\theta}{w} - \frac{v}{w}e^{\frac{X}{US}} - \frac{(1-v-w)}{w}e^{\frac{X}{NZ}} \quad (6)$$

and

$$e^{\frac{X}{NZ}} = \frac{\theta}{(1-v-w)} - \frac{v}{(1-v-w)}e^{\frac{X}{US}} - \frac{w}{(1-v-w)}e^{X/AU}. \quad (7)$$

Substituting (1) and (3) into (6), we obtain

$$e^{X/AU} = \theta - ve^{AU/US} - (1-v-w)e^{AU/NZ}. \quad (8)$$

While substituting (2) and (4) into (7),

$$e^{\frac{X}{NZ}} = \theta - ve^{\frac{NZ}{US}} + we^{AU/NZ}. \quad (9)$$

Optimal basket weights

Let us now assume that the main policy objective of Pacific governments when managing their exchange rate policy is to minimise exchange rate volatility against the US dollar as well as minimise GDP volatility. The government objective function can then be written as follows:

$$Min\ z_1\left(e^{X/_{US}} - \overline{e}^{X/_{US}}\right)^2 + z_2(y-\overline{y})^2, \quad (10)$$

where $0 \le z_1 + z_2 \le 1$.

z_1, and z_2 denote the weight that a government can attribute to the stability of the domestic currency to the US dollar and to output stability, respectively. (The model can be extended to included additional objectives such as stabilising volatility towards the Australian dollar or the New Zealand dollar. For the sake of simplicity, we include only the US dollar in the objective function).

We further assume that GDP fluctuations are a function of deviations of the interest rate (r), of government spending (G), and of the current account (CA) from their initial targets (denoted by $\overline{r}$, $\overline{G}$, and $\overline{CA}$, respectively). The fluctuations in the current account come from the fluctuations of the balance of trade and the revenues from tourism. These relationships can be expressed as follows:

$$(y-\overline{y}) = c_0 + c_1(r-\overline{r}) + c_2(G-\overline{G}) + c_3(CA-\overline{CA}), \quad (11)$$

$$(CA-\overline{CA}) = (BT-\overline{BT}) + (TOUR-\overline{TOUR}), \quad (12)$$

where $c_2, c_3 > 0$ and $c_1 < 0$.

We assume that fluctuations of the balance of trade are a function of the fluctuations of GDP (which represents fluctuations of domestic demand and production capability) and of exchange rates in terms of the US dollar, the Australian dollar, and the New Zealand dollar. The fluctuation of revenues from tourism inflow, however, is a function of exchange rates and demand from the rest of the world. These assumptions take the following mathematical forms:

5 The parameter θ is a number that can take any value. It is the nominal unit value of the home currency.

$$\begin{aligned}(BT - \overline{BT}) = a_0 + a_1(y - \overline{y}) \\ + a_2\left(e^{X/US} - \overline{e}^{X/US}\right) \\ + a_3\left(e^{X/AU} - \overline{e}^{X/AU}\right) \\ + a_4\left(e^{X/NZ} - \overline{e}^{X/NZ}\right),\end{aligned} \tag{13}$$

$$\begin{aligned}(\mathrm{TOUR} - \overline{\mathrm{TOUR}}) = \mathrm{b}_0 + \mathrm{b}_1\left(\mathrm{e}^{X/US} - \overline{e}^{X/US}\right) \\ + \mathrm{b}_2\left(\mathrm{e}^{X/AU} - \overline{e}^{X/AU}\right) \\ + \mathrm{b}_3\left(\mathrm{e}^{X/NZ} - \overline{e}^{X/NZ}\right) \\ + \mathrm{b}_4\left(\mathrm{Y_w} - \overline{\mathrm{Y_w}}\right).\end{aligned} \tag{14}$$

Using Eqns (12)–(14), the output gap can therefore be rewritten as

$$\begin{aligned}y - \overline{y} = c_0 + c_1(r - \overline{r}) + c_2(G - \overline{G}) \\ + c_3\{a_0 + b_0 + a_1(y - \overline{y}) \\ + (a_2 + b_1)\left(e^{\frac{X}{US}} - \overline{e}^{\frac{X}{US}}\right) \\ + (a_3 + b_2)\left(e^{\frac{X}{AU}} - \overline{e}^{\frac{X}{AU}}\right) \\ + (a_4 + b_3)\left(e^{\frac{X}{NZ}} - \overline{e}^{\frac{X}{NZ}}\right) \\ + b_4(Y_w - \overline{Y_w})\}.\end{aligned} \tag{15}$$

Further, the exchange rate gap is a function of domestic and external output gaps and also of the fluctuations of other exchange rates.

$$\begin{aligned}\left(e^{X/us} - \overline{e}^{X/us}\right) = d_0 + d_1(y - \overline{y}) \\ + d_2\left(e^{X/AU} - \overline{e}^{X/AU}\right) \\ + d_3\left(e^{X/NZ} - \overline{e}^{X/NZ}\right) \\ + d_4(Y_w - \overline{Y_w})\end{aligned} \tag{16}$$

Substituting (8) and (9) into (15) and (16) and solving them simultaneously, we obtain the reduced forms:

$$\begin{aligned}(y - \overline{y}) = f_0 + f_1(r - \overline{r}) + f_2(G - \overline{G}) \\ + f_3\theta + f_4(v)\left(e^{\frac{AU}{US}} - \overline{e}^{\frac{AU}{US}}\right) \\ + f_5(v)\left(e^{\frac{NZ}{US}} - \overline{e}^{\frac{NZ}{US}}\right) \\ + f_6(v, w)\left(e^{AU/NZ} - \overline{e}^{AU/NZ}\right) \\ + f_7(Y_w - \overline{Y_w})\end{aligned} \tag{17}$$

and

$$\begin{aligned}\left(e^{X/us} - \overline{e}^{X/us}\right) = g_0 + g_1(r - \overline{r}) \\ + g_2(G - \overline{G}) + g_3\theta \\ + g_4(v)\left(e^{\frac{AU}{US}} - \overline{e}^{\frac{AU}{US}}\right) \\ + g_5(v)\left(e^{\frac{NZ}{US}} - \overline{e}^{\frac{NZ}{US}}\right) \\ + g_6(v, w)\left(e^{AU/NZ} - \overline{e}^{AU/NZ}\right) \\ + g_7(Y_w - \overline{Y_w}).\end{aligned} \tag{18}$$

(The full expressions of $f_0 - f_7$ and $g_0 - g_7$ can be found in the Appendix.)

To minimise the loss function, we derive the function with respect to v and w:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial v} = 2z_1\left(e^{X/us} - \overline{e}^{X/us}\right)\frac{\partial\left(e^{X/us} - \overline{e}^{X/us}\right)}{\partial v} \\ + 2z_2(y - \overline{y})\frac{\partial(y - \overline{y})}{\partial v} = 0,\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial w} = 2z_1\left(e^{X/us} - \overline{e}^{X/us}\right)\frac{\partial\left(e^{X/us} - \overline{e}^{X/us}\right)}{\partial w} \\ + 2z_2(y - \overline{y})\frac{\partial(y - \overline{y})}{\partial w} = 0.\end{aligned} \tag{20}$$

The partial differential $\frac{\partial\left(e^{X/us} - \overline{e}^{X/us}\right)}{\partial v}$, $\frac{\partial(y-\overline{y})}{\partial v}$, $\frac{\partial\left(e^{X/us} - \overline{e}^{X/us}\right)}{\partial w}$, $\frac{\partial(y-\overline{y})}{\partial w}$ will give us the following constants:

$$\frac{\partial\left(e^{x}/_{us} - \bar{e}^{x}/_{us}\right)}{\partial v} = \frac{\partial g_4(v)}{\partial v} + \frac{\partial g_5(v)}{\partial v} + \frac{\partial g_6(v,w)}{\partial v} = h_1, \quad (21)$$

$$\frac{\partial(y-\bar{y})}{\partial v} = \frac{\partial f_4(v)}{\partial v} + \frac{\partial f_5(v)}{\partial v} + \frac{\partial f_6(v,w)}{\partial v} = h_2, \quad (22)$$

$$\frac{\partial\left(e^{x}/_{us} - \bar{e}^{x}/_{us}\right)}{\partial w} = \frac{\partial g_6(v,w)}{\partial w} = h_3, \quad (23)$$

$$\frac{\partial(y-\bar{y})}{\partial w} = \frac{\partial f_6(v,w)}{\partial w} = h_4. \quad (24)$$

Thus rearranging (19) and (20), we obtain

$$z_1\left(e^{x}/_{us} - \bar{e}^{x}/_{us}\right)h_1 = z_2(y-\bar{y})h_2, \quad (25)$$

$$z_1\left(e^{x}/_{us} - \bar{e}^{x}/_{us}\right)h_3 = z_2(y-\bar{y})h_4. \quad (26)$$

Solving for (25) and (26) simultaneously using Cramer's rule, we obtain

$$v = \frac{qk - nl}{qm - pn} \quad (27)$$

and

$$w = \frac{pk - ml}{pn - qm}. \quad (28)$$

v and w thus give us the optimal weights. (All elements of the right hand side can be found in the Appendix.)

Simulation of the theoretically optimal shares

In this section, we report the results of a simple simulation using the aforementioned model to determine the optimal shares of foreign currencies in the currency baskets of the four countries. In order to calculate the optimal shares as given by Eqns (27) and (28), we first needed to estimate Eqns (11), (13), and (14).

The necessary data for the estimations were collected in the following way. First, we gathered yearly data from the World Bank (2014) database and Asian Development Bank Outlook (2014) for each country for the period 1995 to 2012. The data collected was GDP, nominal interest rate, government expenditure, balance of trade, revenue from tourism, world GDP, the average value of the Australian dollar–US dollar exchange rate, and the average value of domestic currency to the Australian dollar and the US dollar exchange rate. We added revenues from tourism and the balance of trade to obtain the value of the current account. The exchange rate variables were converted into their logarithmic form. As usually performed, we used the trends from the Hodrick–Prescott filter as the expected value of each variable. We then measured the deviation of each variable by deducting the actual value from the estimated value of the respective variable.

We estimated Eqns (13) and (14) simultaneously using two-stage least squares regression, whereas the coefficients in Eqn (11) were obtained using a simple ordinary least squares (OLS) regression. The results are shown in Tables 2–4.

Finally, in order to determine the weights, we need to assign a value to θ. As explained earlier, θ can be any number as it is basically the nominal value of a currency. As the countries do not publish the value of θ, we assume for the sake of simplification that it corresponds to the values we estimate for the *de facto* weights applying the method introduced by Frankel and Wei (1994) in the next section. Table 5 shows the value for v and w as well as θ for all four countries.

Assuming these values for θ, we can make use of all coefficients estimated previously to calculate the optimal value of v and w given the loss function as defined in Eqn (10). The estimated optimal basket weights are listed in Table 6.

According to our estimations, the US dollar should have become the dominant currency in

Table 2
Coefficients of TSLS regression of the deviation of GDP of the domestic to US dollar exchange rate, and of the domestic to Australian dollar exchange rate on the deviation of the balance of trade of Fiji, Samoa, Solomon Islands, and Tonga (Eqn (8))

	FIJ	SOL	TON	SAM
$(y-\bar{y})$	−0.83 (−2.02)[3]	0.65 (1.72)	−0.95[1] (−3.40)	1.51 (0.73)
$(e^{X/\$}-\bar{e}^{X/\$})$	−1.15E + 09 (−1.24)	2.28E + 08 (0.89)	−1.96E + 08[2] (−2.66)	−1.72E + 09 (−1.22)
$(e^{X/A\$}-\bar{e}^{X/\$})$	9.38E + 08 (1.73)	−2.30E + 07 (−0.79)	8.83E + 07 (1.35)	−1.47E + 09 (−0.84)
$(e^{X/NZ\$}-\bar{e}^{X/NZ\$})$	1.63E + 08[1](0.26)	2.17E + 08 (0.89)	−8.86E + 06 (−0.20)	−5.01E + 08 (−0.73)
R^2	0.77	0.23	0.43	0.30

[1]Significant at 1% level.
[2]Significant at 5% level.
[3]Significant at 10% level.
Note: Numbers in parentheses denote *t*-values.
Source: Authors' estimates.
FIJ = Fiji; SOL = Solomon Islands; TON = Tonga; SAM = Samoa; TSLS = two-stage least squares.

Table 3
Coefficients of OLS regression of the deviation of the domestic to US dollar exchange rate, of the domestic to Australian dollar exchange rate, and of world GDP on the deviation of tourism revenue of Fiji, Samoa, Solomon Islands, and Tonga (Eqn (9))

	FIJ	SOL	TON	SAM
$(e^{X/\$}-\bar{e}^{X/\$})$	−9.75E + 08 (−0.57)	−2.13E + 07 (−0.65)	−1.44E + 07[3](−1.86)	−4.28E + 07 (−1.75)
$(e^{X/A\$}-\bar{e}^{X/A\$})$	−5.63E + 08[2](−1.97)	1.00E + 08[3] (0.33)	−3.63E + 06[2](−2.31)	−1.35E + 07 (−0.54)
$(e^{X/NZ\$}-\bar{e}^{X/NZ\$})$	−3.50E + 08[b] (0.96)	−8.56E + 07[3] (−2.07)	2.18E + 06[3](−0.81)	−2.23E + 07 (−1.03)
$(Y_w-\overline{Y_w})$	1.46E − 05 (2.91)	1.67E − 06[3] (2.00)	2.86E−07 (1.11)	1.80E − 06[1] (6.12)
R^2	0.78	0.49	0.43	0.83

[1]Significant at 1% level.
[2]Significant at 5% level.
[3]Significant at 10% level.
Note: Numbers in parentheses denote *t*-values.
Source: Authors' estimates.
ordinary least squares; FIJ = Fiji; SOL = Solomon Islands; TON = Tonga; SAM = Samoa.

all four basket currencies. For Fiji, the model predicts that the optimal share of the US dollar should be close to 50 per cent and 30 per cent for the Australian dollar. For the Solomon Islands, the weight of the US dollar in the basket should be close to 100 per cent. For Tonga, the model predicts a weight of almost two-thirds for the US dollar. Finally, in Samoa the optimal share of the US dollar is around 50 per cent. The model thus provides a useful tool for monetary policy authorities to calculate the optimal shares of foreign currencies in their basket given a specific policy objective.

It has to be noted that these shares depend on the loss function of the government. As we do not know the loss function of the government, we cannot claim that these basket weights are optimal for the countries. Each government needs to decide on its own objective and determine the optimal weights accordingly.

Table 4
Coefficients of TSLS regression of the deviations of interest rate, government spending, and current account on the deviation of GDP of Fiji, Samoa, Solomon Islands, and Tonga (Eqn (6))

	FIJ	SOL	TON	SAM
$(r - \bar{r})$	2.38E + 06 (0.20)	−4.91E + 05 (−0.17)	1.06E + 05 (0.09)	−2.76E + 05 (−0.11)
$(G - \overline{G})$	0.03[1] (4.76)	0.02[1] (3.24)	0.03[1] (4.45)	0.001 (1.15)
$(CA - \overline{CA})$	−0.96 (−1.50)	−0.68 (−1.09)	1.33[3] (2.10)	0.43[2] (2.30)
R^2	0.71	0.49	0.61	0.014

[1]Significant at 1% level.
[2]Significant at 5% level.
[3]Significant at 10% level.
Note: Numbers in parentheses denote *t*-values;
Source: Authors' estimates.
FIJ = Fiji; SOL = Solomon Islands; TON = Tonga; SAM = Samoa; TSLS = two-stage least squares.

Table 5
Estimated values of basket currencies in Fiji, Solomon Islands, Tonga, and Samoa

	F$	SI$	T$	ST
v	0.37	0.98	0.55	0.41
w	0.24	0.03	0.19	0.26
θ	0.52	1.65	0.80	0.77

Source: Authors' estimates.
F$ = Fiji dollar; SI$ = Solomon Islands dollar; ST = tala; T$ = pa'anga.

Table 6
Estimated values of basket currencies in Fiji, Solomon Islands, Tonga, and Samoa

	F$	SI$	T$	ST
v	0.482	0.946	0.648	0.517
w	0.303	0.032	0.192	0.269

Source: Authors' estimates.
F$ = Fiji dollar; SI$ = Solomon Islands dollar; ST = tala; T $ = pa'anga.

Estimating *de facto* basket weights

Ordinary least squares regressions

In this section, we seek to better understand how monetary policy is actually undertaken in the four countries and compare it to our theoretical findings previously. The four countries with currency baskets do not typically publish information about which foreign currencies are in the basket nor their weights. Following the methodology introduced by Frankel and Wei (1994), one can estimate the *de facto* weight of the basket currencies or test whether other exchange rate arrangements follow basket currencies. The basic idea behind Frankel and Wei (1994) is that, in the case of a perfect basket peg, a simple OLS regression analysis on the daily exchange rates against a numeraire currency will uncover the weights in the basket. In other words, the volatility of the Pacific currencies against the numeraire can be explained by the volatility of one or several potential anchor currencies. The estimation takes the following form:

$$e_{i,t} = \alpha_i + \sum_{j=1}^{N} \beta_{i,j} e_{j,t} + u_{i,t}. \tag{29}$$

In Eqn (29), the differences of the logged exchange rates of the daily bilateral exchange rates of each Pacific DMC and other currencies in the basket are expressed by e_i and e_j, respectively, while N denotes the number of different currencies in the basket.

We assume that the four Pacific countries might use the following currencies in their monetary baskets: the Australian dollar, the euro, the New Zealand dollar, the pound sterling, the US

Table 7
Estimates of the currency weights in the currency baskets of Fiji, Samoa, Solomon Islands, and Tonga (direct OLS)

	Fiji	Solomon Islands	Tonga	Samoa
	F$	SI$	T$	ST
A$	0.178[1] (9.26)	0.027[1] (3.03)	0.183[1] (11.41)	0.211[1] (24.20)
€	0.200[1] (11.11)	–	–	–
£	–	–	0.065[1] (4.38)	0.064[1] (8.22)
¥	0.022 (1.57)	0.013 (1.09)	–	0.004[1] (0.74)
W	–	–	–	0.026[1] (3.76)
NZ$	0.297[1] (16.15)	–	0.214[1] (13.82)	0.296[1] (36.40)
$	0.304[1] (18.88)	0.960[1] (68.31)	0.537[1] (41.11)	0.400[1] (49.79)
R^2	0.54	0.70	0.58	0.84

[1]Significant at 1% level.
[2]Significant at 5% level.
[3]Significant at 10% level. The coefficient of the constant is not reported.
Note: Numbers in parentheses denote *t*-values;
Source: Authors' estimates.
$ = US dollar; A$ = Australian dollar; € = euro; F$ = Fiji dollar; NZ$ = New Zealand dollar; £ = pound sterling; SI$ = Solomon Islands dollar; ST = tala; T$ = pa'anga; Vt = vatu; W = won; ¥ = yen; ordinary least squares.

dollar, the won, and the yen.[6] The Canadian dollar is used as numeraire to measure the volatility of the exchange rates and thereby identify the basket currencies. For this analysis, we downloaded the daily exchange rates from USForex and Bloomberg from 1 January 2001 to 21 June 2013.

We were able to download daily exchange rate data towards the Canadian dollar for all Pacific countries' currencies except for the pa'anga. We therefore triangulated the data by using the pa'anga–US dollar and US dollar–Canadian dollar exchange rates. Also, there were numerous missing values for the exchange rates of the Solomon Islands dollar–Canadian dollar and tala–Canadian dollar. We thus applied the same approach to proxy the missing values. All data were converted into differenced log forms and smoothed using the Hodrick–Prescott filter.

In the first attempt, we estimated the weight of each currency by pooling all observations and estimating a simple OLS regression with the restrictions that the coefficients are larger than zero and add up to one. The results (Table 7) suggest that Fiji uses a currency basket that consists mainly of the US dollar, New Zealand dollar, euro, and Australian dollar.[7] For the Solomon Islands, the results indicate that the Solomon Islands dollar was almost fully pegged against the US dollar, and these results are supported by other studies, such as Wood (2010). For Tonga, the regression suggests that the pa'anga was pegged mainly to the US dollar and, to a lesser extent (around 20 per cent) to the Australian dollar and New Zealand dollar. According to our estimation, the Samoan tala was oriented towards a very similar currency basket but with a relatively high weight for the New Zealand dollar and a lower weight for the US dollar. For all four countries, our simple pooled regressions explain between 54 and 84 per cent of the variations.

Our results can also be tested through backwards OLS. This means that variables that do not enhance the model fit are eliminated stepwise until the most parsimonious model is reached. In addition, we employ a non-negativity constraint by removing variables

6 We did not include the yuan as a possible foreign currency in the basket because during the period of analysis, the yuan was pegged almost 100 per cent to the US dollar (Yoshino et al. 2014). This peg would cause a multi-collinearity problem in the estimations.
7 The small, statistically significant, but negative results for pound sterling and the won are difficult to interpret.

Table 8
Estimates of the weights in the currency baskets of Fiji, Samoa, Solomon Islands, and Tonga (backwards OLS)

	F$	SI$	T$	ST
A$	0.215[1] (9.47)	0.031[2] (2.23)	0.166[1] (8.65)	0.208[1] (20.54)
€	0.214[1] (12.58)	–	–	–
£	–	–	0.063[1] (4.21)	0.065[1] (8.40)
¥	–	–	–	–
W	–	–	–	0.026[1] (3.76)
NZ$	0.288[1] (15.71)	–	0.217[1] (13.91)	0.296[1] (36.27)
$	0.349[1] (21.81)	0.973[1] (75.52)	0.525[1] (36.50)	0.401[1] (51.46)
R^2	0.54	0.69	0.57	0.84

[1]Significant at 1% level.
[2]Significant at 5% level.
Note: The coefficient of the constant is not reported.
Source: Authors' calculations.
OLS=ordinary least squares.

with negative signs. The estimation results of the backwards OLS regressions are shown in Table 8.

Overall, we see that the R^2s from the backwards OLS approach are very similar to the those from the simple OLS approach, which indicates that we do not lose much information even when eliminating several variables. According to the backwards OLS regressions, in all countries, the US dollar plays the most important role in their currency arrangements. It seems that the two Asian currencies included, namely the yen and the won, have not gained popularity among these currency baskets.

Looking in more detail at Fiji, the regression results show that the Australian dollar and the euro take approximately the same share in the basket—about 21 per cent. The New Zealand dollar has an estimated weight of 29 per cent, whereas the US dollar has a share of about 35 per cent. In the case of the Solomon Islands, we again find that the currency is almost fully pegged against the US dollar with an estimated share of over 97 per cent. Tonga and Samoa seem to conduct very similar exchange rate policies. Their main basket currency appears to be the US dollar, with Tonga having 53 per cent and Samoa 40 per cent. The next two largest shares are the Australian dollar and the New Zealand dollar. Both countries also appear to have a small share of pounds sterling included, whereas the won is only present in Samoa, with a share of 3 per cent.

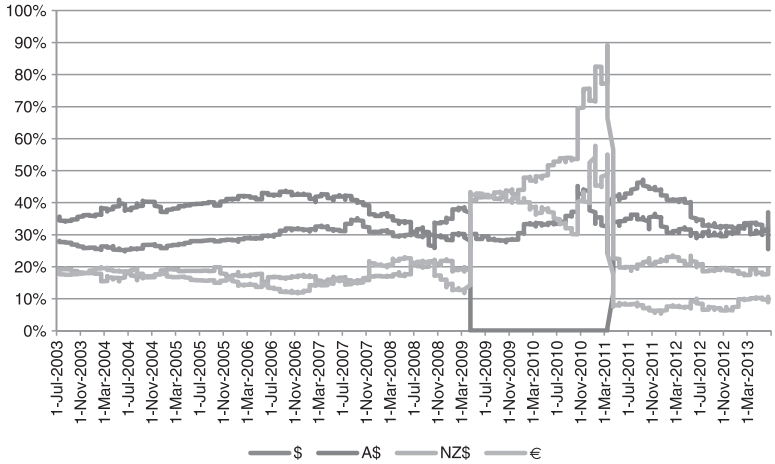
Rolling regression approach

In order to better understand how the exchange rate policies of the four countries change over time, we applied a rolling regression approach covering the period from 2003 to 2013. A rolling regression approach means that we regress the daily exchange rates of the four countries with the other exchange rates in the baskets in separate 480-day-long windows from the beginning until the end of the period under analysis. In total, we calculated 2069 regression coefficients. A rolling regression approach is useful for taking into account that the weights in basket currencies are often calculated using moving averages. For example, the Reserve Bank of Fiji uses a 3-year moving average of trade flows to determine the basket weights. The results of the rolling regressions are summarised in Figures 2–5.[8]

The vertical axis in Figure 2 records the magnitude of the regression coefficients that the OLS estimations yielded for each 480-day period in the case of Fiji. As we move forward in time

8 In order to facilitate the readability of the figures, the statistical significance of the coefficients is not reported nor the coefficients with a value below zero.

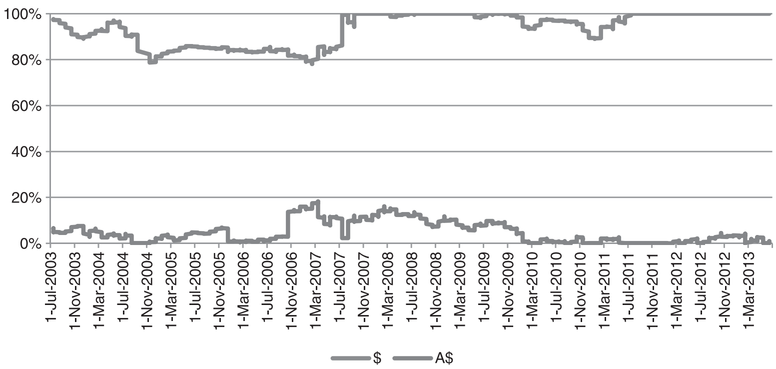
Figure 2
Rolling regressions on Fiji dollar basket currencies



$ = US dollar; A$ = Australian dollar; € = euro; NZ$ = New Zealand dollar.

Source: Authors' estimatess.

Figure 3
Rolling regressions on Solomon Islands dollar basket currencies



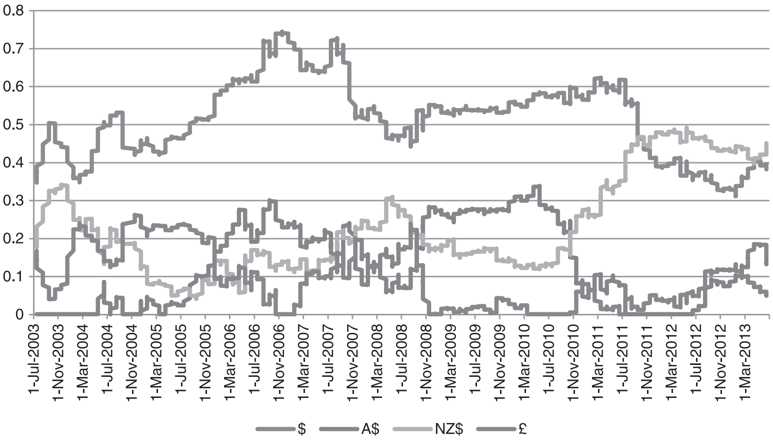
$ = US dollar; A$ = Australian dollar

Source: Authors' estimates.

along the horizontal axis, the coefficients of the subsequent 480-day period are added. We observe that the weights of the four currencies in the basket changed constantly and, during several periods, rather sharply. From 2003 to mid-2008, the Australian dollar had the largest weight. Then, the US dollar became the largest share. During the global financial and economic crisis, the New Zealand dollar became increasingly important as an anchor. After the financial crisis, the US dollar remained the dominant currency in the basket.

Jayaraman and Narayan (2011) provide an insightful analysis of the monetary policy of Fiji during and after the global financial crisis. They describe how Fiji's monetary authorities became increasingly worried about the negative impact on Fiji's tourism and commodity exports

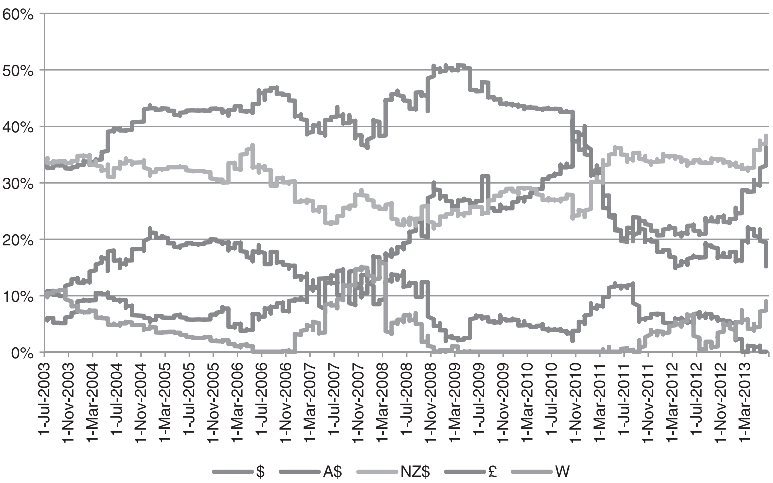
Figure 4
Rolling regressions on Pa'anga basket currencies



$ = US dollar; A$ = Australian dollar; NZ$ = New Zealand dollar; £ = pound sterling.

Source: Authors' estimates.

Figure 5
Rolling regressions on Tala basket currencies



$ = US dollar; £ = pound sterling; A$ = Australian dollar; NZ$ - New Zealand dollar; W = won.

Source: Authors' calculations.

and decided to devalue the Fiji dollar by 20 per cent in April 2009. Our rolling regression estimates show that the devaluation meant that the weight of the Australian dollar fell to zero, while the shares of the Euro and New Zealand dollar increased sharply. The Australian dollar was reintroduced in the basket in early 2011 when the downside risks of the world economy had become smaller.

Figure 3 shows the results of the rolling regression on the Solomon Islands dollar. We observe that from 2003 to 2011, the Australian dollar regularly reached weights of above 5 per cent. However, since then, it has lost its importance and the Solomon Islands dollar seems to be pegged almost 100 per cent to the US dollar. This observation confirms earlier findings by Jayaraman and Narayan (2011) who describe the exchange rate regime of Solomon Islands as a *de facto* peg to the US dollar.

The currencies in Tonga's basket changed substantially over the time period, as we can observe in Figure 4. Overall, it seems that the US dollar had the largest weight in almost every period except from mid-2011 onward. The New Zealand dollar and Australian dollar come next in importance. The New Zealand dollar gained in popularity, especially from mid-2011 onward, and accounts for the largest weight in the recent periods of our sample. In contrast, the pound sterling seem to have lost importance and had a share of about 5 per cent at the end of the period.

Figure 5 shows that the situation in Samoa has been similar to that of Tonga; the weights of the main basket currencies appear to have varied greatly over time. The US dollar has been the most important anchor in the basket; however, its share declined below the share of the New Zealand dollar during the global financial and economic crisis. The New Zealand dollar's share fluctuated as well but remained more stable at around 30 per cent. The share of the Australian dollar increased until the end of 2010 but declined towards the end of the period under analysis. This lower weight could be a reaction to the fact that the share of Australia in Samoa's exports fell from over 68 per cent in 2008 to 38 per cent in 2013.

Conclusions

We studied the exchange rate choices of the four Pacific DMCs that employ a basket currency policy in managing their exchange rate. We first built a simple theoretical model to analyse an optimal basket currency policy. We adopted the model introduced by Yoshino et al. (2003) but made two important extensions. First, we extended the model to the case of four countries.[9] Second, we modelled tourism flows. Solving the model, we were able to calculate the theoretically-optimal weights of up to three foreign currencies in the basket. Applying the model to the four country cases, we undertook simulations given the objective function of minimising fluctuations of the exchange rate against the US dollar. The simulations show how the model can be used to determine the optimal currency basket weights given a certain policy objective, such as to stabilise GDP or exchange rate fluctuations.

We then estimated the *de facto* weights of the basket currencies following the methodology introduced by Frankel and Wei (1994). We found that the US dollar still holds the major share in the basket currencies of the four countries. Other currencies that have significant shares in the currency baskets were the Australian dollar, the New Zealand dollar, the euro, the pound sterling, and the won. We also conducted a rolling regression approach to analyse the change in the basket composition in a more frequent manner. The rolling regression results indicated that the monetary authorities in Fiji maintained relatively constant shares in their currency portfolios, except in the aftermath of the global financial crisis when the Fiji Dollar was devalued. In the case of Solomon Islands, the monetary authorities have adopted an almost full peg to the US dollar. The weights in the currency baskets of Samoa and Tonga were often reviewed in order to adapt to changing trade patterns.

When comparing the predictions of optimal weights based on the model, we find that given the assumption of minimising exchange rate fluctuations to the US dollar, the Pacific countries should increase the weight of the US dollar in their baskets. However, as stated previously, if the objective function of the government is different, this result will also change. Another caveat is that the empirical work in The Optimal Basket Weights section was only able to use annual data for several key macro-economic indicators (such as GDP). Given the relatively small number of observations, the precision of the model is also limited. The monetary authorities in the Pacific might

9 A more complicated approach would be to use a dynamic model as, for example, in Yoshino et al. (2015, 2014).

have more frequent data of key macroeconomic variables, which would enable them to calculate more accurate weights. Finally, we need to recall that improved exchange rate policy is a necessary but certainly not a sufficient condition for successful economic development of the four economies covered. All four economies are small, remote, and disproportionately sensitive to external economic shocks. However, we hope that the model is a valuable contribution to the exchange rate policy of these countries. Other countries with basket currency arrangements in place might also implement the model.

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Appendix

Coefficients used in the model

$$f_0 = \frac{c_0 + c_3a_0 + b_0\,c_3 + d_0c_3(a_2 + b_1\,)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$f_1 = \frac{c_1}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$f_2 = \frac{c_2}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$f_3 = \frac{c_3((a_2 + b_1\,)(d_2 + d_3) + a_3 + b_2 + a_4 + b_3)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$f_4(v) = -\frac{c_3(d_2(a_2 + b_1\,) + a_3 + b_2)v}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$f_5(v) = -\frac{c_3(d_3(a_2 + b_1\,) + a_4 + b_3)v}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$f_7 = \frac{c_3((a_2 + b_1\,)d_4 + b_4)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$g_0 = d_0 + d_1f_0$$

$$g_1 = d_1f_1$$

$$g_2 = d_1f_2$$

$$g_3 = d_1f_3 + d_2 + d_3$$

$$g_4(v) = d_1f_4(v) - d_2v$$

$$g_5(v) = d_1f_5(v) - d_3v$$

$$g_6(v, w) = d_1f_6(v, w) - d_2(1 - v - w) + d_3w$$

$$g_7 = d_1f_7 + d_4$$

$$f_6(v, w) = \frac{c_3\{[(d_2 + d_3)(a_2 + b_1\,) + a_4 + b_3 + a_3 + b_2]w - [d_2(a_2 + b_1\,) + a_3 + b_2](1 - v)\}}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}$$

$$\begin{aligned} k = {} & z_2h_2\left(f_0 + f_1(r - \bar{r}) + f_2\left(G - \overline{G}\right) + f_3\theta + f_7\left(Y_w - \overline{Y_w}\right)\right) - z_1h_1\left(g_0 + g_1(r - \bar{r}) + g_2\left(G - \overline{G}\right) + g_3\theta + g_7\left(Y_w - \overline{Y_w}\right)\right) \\ & + \left(d_2z_1h_1 + (d_1z_1h_1 - z_2h_2)\frac{c_3(d_2(a_2 + b_1\,) + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\,)}\right)\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right) \end{aligned}$$

$$
\begin{aligned}
l = {} & z_2h_4\left(f_0 + f_1(r - \bar{r}) + f_2\left(G - \overline{G}\right) + f_3\theta + f_7\left(Y_w - \overline{Y_w}\right)\right) \\
& -z_1h_3\left(g_0 + g_1(r - \bar{r}) + g_2\left(G - \overline{G}\right) + g_3\theta + g_7\left(Y_w - \overline{Y_w}\right)\right) + d_2z_1h_3\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right) \\
& +(d_1z_1h_3 - z_2h_4)\frac{c_3(d_2(a_2 + b_1\ ) + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right)
\end{aligned}
$$

$$
\begin{aligned}
m = {} & \left(d_2z_1h_1 + (d_1z_1h_1 - z_2h_2)\frac{c_3(d_2(a_2 + b_1\ ) + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right) \\
& -\left(d_2z_1h_1 + (d_1z_1h_1 - z_2h_2)\frac{c_3(d_2(a_2 + b_1\ ) + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{AU}{US}} - \bar{e}^{\frac{AU}{US}}\right) \\
& -\left(d_3z_1h_1 + (d_1z_1h_1 - z_2h_2)\frac{c_3(d_3(a_2 + b_1\ ) + a_4 + b_3)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{NZ}{US}} - \bar{e}^{\frac{NZ}{US}}\right)
\end{aligned}
$$

$$
n = \left((d_2 + d_3)z_1h_1 + (d_1z_1h_1 - z_2h_2)\frac{c_3((d_2 + d_3)(a_2 + b_1\ ) + a_4 + b_3 + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right)
$$

$$
\begin{aligned}
p = {} & \left(d_2z_1h_3 + (d_1z_1h_3 - z_2h_4)\frac{c_3(d_2(a_2 + b_1\ ) + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right) \\
& -\left(d_2z_1h_3 + (d_1z_1h_3 - z_2h_4)\frac{c_3(d_2(a_2 + b_1\ ) + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{AU}{US}} - \bar{e}^{\frac{AU}{US}}\right) \\
& -\left(d_3z_1h_3 + (d_1z_1h_3 - z_2h_4)\frac{c_3(d_3(a_2 + b_1\ ) + a_4 + b_3)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{NZ}{US}} - \bar{e}^{\frac{NZ}{US}}\right)
\end{aligned}
$$

$$
q = \left((d_2 + d_3)z_1h_3 + (d_1z_1h_3 - z_2h_4)\frac{c_3((d_2 + d_3)(a_2 + b_1\ ) + a_4 + b_3 + a_3 + b_2)}{1 - c_3a_1 - d_1c_3(a_2 + b_1\ )}\right)\left(e^{\frac{AU}{NZ}} - \bar{e}^{\frac{AU}{NZ}}\right)
$$