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Student teachers' mathematics attitudes, authentic investigations and use of metacognitive tools

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Abstract Based on findings from a semester-long study, this article examines the development of Samoan prospective teachers' mathematical understandings and mathematics attitudes when investigating authentic contexts and applying working mathematically processes, mental computations and problem-solving strategies to find solutions of problems. The prospective teachers had enrolled for the *second* time (having failed their first attempt), in the first-year mathematics methods course of a 2-year Diploma of Education (Primary) programme. The group also included those enrolled in the Diploma of Education (Early Childhood and Special Needs) programmes, who recognizing their own limited understanding of mathematics would ordinarily shy away from opportunities for improvement. Given the negative mathematical and learning experiences, this group was ideal to engage in innovative and creative approaches that would make mathematics learning more meaningful and contextual in a Samoan environment. Only data from the attitudinal questionnaires and interviews are presented in this article. Main findings have implications for teaching and learning mathematics.

Keywords Attitudinal change · Authentic investigation · Metacognitive tools (concept maps and vee diagrams) · Working mathematically · Problem-solving strategies

Introduction

In some universities, pre-service programmes have minimum entry requirements for mathematics, which usually are passes in a mathematics course in the last 2 years of

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secondary schooling (Ryan and McCrae 2005/2006). For the prospective primary teachers (PPTs) enrolled in the Diploma of Education (Primary) programme at the National University of Samoa, the entry requirement is either a Foundation Education Certificate of Attainment[1] or matured age entry with relevant work experience. In meeting the criterion for entry, it was not mandatory for PPTs to have passed a mathematics course in the last 2 years of secondary schooling, i.e. Year 12[2] or Year 13.[3] As a consequence, it was foreseeable that gaps in mathematics content knowledge were inevitable.

This article is part of a larger study that investigated a group of PPTs' creative use of authentic contexts for mathematical investigations of working mathematically processes, mental computations and problem-solving strategies. Based on the maths ideas and unit concepts in the new 2013 Samoa Primary Mathematics Curriculum (herein referred to as the new Curriculum) (Samoa Ministry of Education Sports and Culture [SMESC] 2013), the innovative application of two metacognitive tools, concepts maps and vee diagrams, was used to redefine and illustrate understanding of solution methods to mathematical problems. Working mathematically processes as defined in the new Curriculum include five interrelated sets of processes: (1) interpreting and/or posing questions, (2) strategically thinking and representing, (3) reasoning and justifying, (4) reflecting and evaluating and (5) communicating mathematically. It is envisaged in the new Curriculum that the implementation of these processes as routine classroom practice would over time provide teachers and students with the language to co-construct the developmental aspects of doing, learning and understanding mathematics.

The authors were also interested in assessing the impact of using the innovative tools, mental computations and creative authentic investigations on PPTs attitudes towards mathematics. The metacognitive tools and mental computations are labelled 'innovative' as PPTs were using the tools and mental strategies for the first time. Similarly, the use of authentic contexts is labelled 'creative' given the first-time experience for PPTs to choose an authentic context as a basis for an investigation for mathematical ideas and application of working mathematically processes and problem-solving strategies based on content areas of the new Curriculum.

This article presents only the attitudinal and interview data of the larger study to answer the following focus questions: (1) What were the primary prospective teachers' general attitudes to mathematics at the *beginning* and *end* of the one-semester mathematics methods course? (2) What factors appeared most influential in changing primary prospective teachers' attitudes towards mathematics?

The following sections provide the theoretical frameworks, context and background of the study including a review of the relevant literature, methodology, discussion, main findings and implications for the teaching and learning of mathematics.

Theoretical frameworks

Teacher knowledge

Subject matter knowledge for teaching as initially proposed by Shulman (1986, 1987) consisted of three categories: (1) *content knowledge*, which includes knowledge of the

[1] A 1-year post-secondary programme required for entry into the Diploma of Education Program.

[2] Equivalent to Year 10 Australian system or Year 11 NZ system.

[3] Equivalent to Year 11 Australian system or Year 12 NZ system.

domain, its organizing and conceptual structure, and processes of validation and production; (2) *curriculum knowledge* of all curricular and programme documentations, important for teaching the subject across levels (i.e. vertical curriculum knowledge) and knowledge of the curriculum that students are learning in other subject areas (i.e. lateral curriculum knowledge) and (3) *pedagogical content knowledge*.

Specifically, the term '*pedagogical content knowledge*' comprises teachers' knowledge of the types of topics students usually find difficult, the nature of these difficulties and the 'most useful forms of representation of those ideas … (and) the most useful ways of representing, formulating the subject to make it comprehensible to others' (Shulman 1986, p. 9). Some researchers (e.g. Ball and Bass 2000; Hill et al. 2005) subsequently proposed 'mathematics knowledge for teaching' as a specific form of mathematical knowledge used to carry out the *work of teaching*. This knowledge is concerned with the tasks involved in teaching and the mathematical demands of these tasks. According to Kilpatrick et al. (2001), each of these tasks involves knowledge of mathematical ideas, skills of mathematical reasoning, fluency with examples and terms, and thoughtfulness about the nature of mathematical proficiency. The findings from Hill et al. (2005) further inform that teachers' content knowledge should be at least content-specific and even better specific to the knowledge used in teaching children. Ball and her colleagues (Hill et al. 2008; Ball et al. 2008) further elaborated on 'mathematics knowledge for teaching' as referring to such knowledge as *specialized content knowledge*, i.e. subject matter knowledge and skill unique to teaching as opposed to common content knowledge which is the mathematical knowledge and skill used in non-teaching settings. Specialized content knowledge was defined as 'the mathematical knowledge that allows teachers to engage in particular *teaching* tasks' such as following students' mathematical thinking, evaluating the validity of student-generated strategies and making sense of a range of student-generated solution paths (Hill et al. 2008, p. 377). Ball et al. (2008) further specified different types of pedagogical content knowledge, such as intersections between knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum (p. 403).

In our larger study, we argued that for specialized content knowledge to work effectively, primary teachers needed to be competent with the content of the curriculum that they planned to teach. Also, Ball et al. (2008) emphasized that '(t)eachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content' (p. 404).

Unlike the studies by Hill et al. (2005, 2008) and Ball et al. (2008), which focused on mathematical knowledge for certified teachers already teaching, our larger study focused only on assessing Samoan PPTs' content-specific knowledge and ability to solve items based on this content (not included here) and their developing attitudes towards mathematics as part of their Diploma programme. In this article, only the attitudinal data are presented to answer the focus questions aforementioned. Being mathematically competent with SMESC's Primary and Early Secondary Mathematics curricula (SMESC 2003, 2013) and having a positive attitude towards mathematics was conceptualized as two of several factors influencing teachers' goals and plans, reflections and enactment of teaching in Samoan classrooms.

Meaningful learning

During the pre-service programme, PPTs are themselves learners of the relevant content knowledge and pedagogical content knowledge of the discipline. With this in mind, a constructivist theoretical approach was used in the study to examine the ways PPTs built on their cognitive structures, broadened and developed deep understanding of mathematics whilst using concept maps, vee diagrams and authentic contexts. Generally and in the context of the larger study reported here, the constructivist perspective advocates *meaningful learning* as learning in which PPTs are actively engaged with the construction of their own meanings and subsequent communication of these for public scrutiny and assessment as learners of mathematics (Dewey 1938; Vygotsky 1978). In this regard, Ausubel's cognitive theory (2000) is also relevant, particularly as it similarly conceptualizes meaningful learning as deliberate connections between new knowledge and existing knowledge resulting in the PPTs reorganization of cognitive structures to assimilate and/or accommodate this new experience (i.e. *thinking*) in creating meaning for themselves.

According to Gowin's (1981) educating theory, 'to teach is to try to change the meanings of students' experience, and students must grasp the meaning before they deliberately learn something new. Learning is never entirely cognitive. Feelings accompany any thinking that moves to reorganize meaning. In educating we are concerned to integrate thinking, feeling, and acting' (Gowin 1981, p. 42). Within this perspective, a powerful moment in educating occurs when the creation/resolution of meaning is accompanied by a 'feeling of significance' (i.e. *feelings*) that the meaning is grasped; this could lead to further actions and choices by the student (i.e. *acting*). Gowin defines 'felt significance' when human feelings merge with meaning, we are able to make sense of an experience. Also, when 'thinking and acting' come into play, the good feeling lasts; feelings are connected with ideas of their significance. Value is felt significance. The construct of both feelings and significance is necessary because we can have one without the other (Gowin 1981, p. 43). This feeling of significance or the connection making, in the educating theory, is the basis of value in experience. When students feel the significance of a learning moment, they are adding value to the experience. 'Value is what holds things together' (p. 45). Thus, the educating theory makes more explicit the roles of: (a) 'feelings' as part of the process of meaningful learning and (b) inherent responsibilities of the learner to learn and the teacher to teach meaningfully. The process of meaningful learning is also described by Piaget's (1972) notion of cognitive disequilibrium, particularly when previous knowledge is challenged by unfamiliar or misunderstood experiences indicating a need to move towards cognitive equilibrium as these experiences are reconceptualized and the knowledge reconstructed to deliberately accommodate meaningfully new knowledge into existing cognitive structures (i.e. *thinking*). Particularly pertinent in the study is Gowin's educating theory (1981) as it provides the most relevant theoretical basis for the epistemological vee diagram used in our study. For the innovative strategies introduced and subsequent encouragement of students' communication of their own thinking and reasoning, Vygotsky's concepts of the zone of proximal development (ZPD) and language use in social communications and interactions (Vygotsky 1978) are considered equally relevant in guiding workshop activities and making sense of students' actions and responses.

Gowin's epistemological vee (a vee shape, see Gowin 1981) explicates the principles of his educating theory and provides a means of guiding the *thinking* and *reflections* involved when one is *making connections* between the *conceptual structure* of a discipline on the one hand and its *methods of inquiry* on the other, as required for the investigation and/or analysis of a phenomenon or event to generate new knowledge claims as answers to some focus questions. To guide the *thinking* and *reasoning* involved in *problem-solving* in mathematics, the original epistemological vee (Gowin 1981) was later modified (Afamasaga-Fuata'i 1998, 2005, 2008). The vee's left side, the '*Thinking*' side, depicts the philosophy or personal beliefs and theoretical framework driving the investigation/analysis of a phenomenon/event to answer some focus questions. On the vee's right side, the '*Doing*' side, are the records, methods of transforming the records to generate some answers or new knowledge claims and value claims.

The social constructivist perspectives view learning as the construction of knowledge to make sense of our experiences whilst at the same time socially interacting with others. In an educational setting, this means students and teachers interact with each other socially as they learn from each other's strategies. The social perspective also views classroom practices as means of reinforcing certain views of what it means to learn and succeed in mathematics. Overall, learning mathematics therefore is viewed as involving both individual and social processes. According to Schoenfeld (1991) and Ernest (1999), when an effort is made to *change* classroom practices into activities that involve: *questioning, analysing, conjecturing, refuting, proving, extending* and *generalizing* as students solve problems, those rituals and practices can actually shape the behaviour and understanding of students *by making it more natural for them to think and reason mathematically*. Collectively, both the meaningful learning and social constructivist perspectives support the metacognitive development of students' understanding and the active construction of mathematical thought whilst publicly presenting, for example, student-constructed concept maps and vee diagrams, within a social setting (Afamasaga-Fuata'i 2009, p. 241).

Mathematics attitudes

Numerous definitions of attitudes abound in the literature. For example, Allport (1935) describes an *attitude* as 'a mental or neural state of readiness, organized through experience, exerting a directive or dynamic influence upon the individual's response to all objects and situations with which it is related' (p. 798). The way in which one behaves, reacts and/or influences objects, issues, people or events is very much determined by their attitudes and beliefs. This view of attitude is also expressed by Hogg and Vaughan (2005) as 'a relatively enduring organization of beliefs, feelings, and behavioral tendencies towards socially significant objects, groups, events or symbols' (p. 150) and by Eagly and Chaiken (1993) as 'a psychological tendency that is expressed by evaluating a particular entity with some degree of favor or disfavor' (p. 1).

According to studies in the learning of mathematics, the attitude and belief of the learner regarding their competency and achievement levels are directly related. For example, Schoenfeld (1989), McLeod (1992) and Broun et al. (1988) found that student performance in mathematics is closely linked to student attitudes, i.e. positive attitudes influence mathematics performance positively whilst negative attitudes influence mathematical performance negatively. In a study conducted by Eleftherios and Theodosios (2007), they found that the factor correlating most positively with performance and mathematical ability was the 'love of mathematics'. This parallels with Schoenfeld's (1989) and Cobb's (1986) findings where it was determined that a positive relationship between beliefs and learning of mathematics exists.

In studying how attitudes are formed, researchers have determined that attitudes are made up of different components: *emotional*, *cognitive* and *behavioural*, and that we can be consciously aware of these attitudes (*explicit attitudes*) or unconsciously aware (*implicit attitudes*). Irrespective of which type, both sets of attitudes will still have an effect on one's behaviour and beliefs. Given this understanding, the experience a learner gains through his/her interaction with the environment is key to developing initial perceptions and more long-lasting attitudes. It is these that will determine learning behaviour (McLeod 2009; Hogg and Vaughan 2005; Eagly and Chaiken 1993; Weber 1992; LaPiere 1934).

A number of classroom studies have been conducted at various levels by numerous mathematics teachers, educators and researchers to address the common societal perception that 'mathematics is difficult'. Whilst these efforts often end up as being unsuccessful and quite difficult due to the already entrenched attitudes and feelings that students have by the time they reach secondary and post-secondary levels, some studies have also recorded some success with the development of positive mathematics attitudes as a result of some innovative and/or creative approaches and strategies at primary levels (Bragg 2007; Nisbet and Williams 2009). Kloosterman and Gorman (1990) suggest that the existence and persistence of the belief that some students learn more readily than others and not everyone will be high achievers in school can lead to a notion that affects achievement in mathematics: the notion that it makes little sense to put forth effort when it does not produce results that are considered desirable. Many other factors have been found in the literature to affect learning and attitude. These include motivation, the quality of instruction, time-on-task, task value and classroom conversations (Woolfolk and Margetts 2007, Hammond and Vincent 1998; Reynolds and Walberg 1992) and as a result of social interactions with their peers (Reynolds and Walberg 1992; Taylor 1992). Woolfolk and Margetts (2007) also noted that students' interest in and enjoyment about what they are learning is one of the most important factors in education in general.

Research of attitudes towards mathematics has concluded that people (children and adults alike) with poor feelings towards mathematics are highly likely to be hindered in terms of engaging with mathematics and therefore the learning of mathematics (Garden 1997; McLeod 1992). Even so, according to the findings by Furinghetti and Morselli (2009), the complexity of mathematical learning from the affective and cognitive perspectives would suggest that although other factors are involved, the relationship is nonetheless symbiotic. In another study by Wilkins et al. (2002), three groups of variables relating to student achievement in mathematics and science were identified: *personal* variables, including affective qualities; *pedagogical* variables, such as the quality of the teaching; and *environmental* variables, including social factors related to home and school. Ercikan et al. (2005) used the same variables in a mathematics study across three countries (viz., USA, Canada and Norway) in which they found that one's attitude towards mathematics was 'the strongest predictor of participation in advanced mathematics courses was students' attitude towards mathematics…' (p. 5).

Many studies have been conducted on mathematics attitudes and teaching (Leder 1987; McLeod 1992; Zan et al. 2006). For our study, McLeod's (1992) attitude definition is used, namely '*affective responses that involve positive or negative feelings of moderate intensity and reasonable stability*' (p. 581). McLeod contends that attitudes develop with time and experience and are reasonably stable, so that hardened changes in students' attitudes may have a long-lasting effect. Lefton (1997) also argues that attitude is a learnt pre-disposition to respond in a consistently favourable or unfavourable manner towards a given object very similar to the perspectives of Eagly and Chaiken (1993) and Hogg and Vaughan (2005). According to Dossey et al. (1988) and others (Woolfolk and Margetts 2007), positive and negative experiences in schools often generate learnt responses which may in turn influence students' attitudes as they get older, when positive attitudes towards mathematics appear to become progressively weak as the mathematics becomes increasingly advanced. Being cognizant of these complex interacting factors was useful in guiding our research study regarding the potential impact of the innovative metacognitive strategies, mental strategies and creative authentic mathematical investigations on PPTs' attitudes, motivation to complete mathematical tasks and subsequent mathematics performance.

Numeracy and mathematical competence

Defining 'numeracy' or 'mathematical literacy' as comprising requisite mathematical skills that 'enable an individual to cope with the practical demands of everyday life' (Cockcroft 1986) and 'data needs of modern life' (Steen 2001), 'mathematics competence' is defined as encompassing the ability and critical problem-solving skills to effectively apply knowledge and skills prescribed by SMESC's Primary and Early Secondary Curriculum. Critical problem-solving skills include the ability to interpret and comprehend mathematical knowledge embodied in various representations, such as the spatial, numeric, algebraic, symbolic, tabular, diagrammatic, graphic and geometric. Collectively, numeracy and mathematics competence are linked to literacy in ways that enhance students' abilities to communicate and negotiate meanings as part of their learning activities. According to Pugalee (1999), four interrelated thinking processes, namely problem-solving, representing, manipulating and reasoning, underpin mathematical literacy or numeracy.

Authentic mathematical investigations

The use of authentic contexts or real-life situations as sites for problem-solving and/or mathematical investigations has been the hallmark of inquiry-based or reformed approaches (NCTM 2007; Baroody and Coslick 1998) to teaching and learning mathematics in order to encourage the development of students' creative thinking and reasoning, application of working mathematically processes, and strategically applying existing knowledge and skills to design and evaluate the investigation and to generate potentially viable solutions (NCTM 2007; AAMT 2007; Romberg 1994). Jaworski (1986) defines mathematical investigations as contextualized problem-solving tasks through which students can speculate, test ideas and argue with others to defend their solutions. In their study of primary students, Diezmann et al. (2001) found that mathematical investigations provided 'the children with opportunities to perform calculations and use mathematical tools in context …. (t)hey also provide(d) a context for children to reason, explain thinking, to justify conclusions and to analyse situations, all indicators of mathematical literacy' (p. 170).

Context and background

Samoan national literacy and numeracy trends

The Samoa Primary English and Literacy Levels test gives national results of literacy and numeracy levels at Year 4 (SPELL I) and Year 6 (SPELL II). Results show that the percentage of 'at-risk' for different Year 4 cohorts over the years has decreased (SMESC 2008); however, a disturbing trend is being observed when 2 years later with the SPELL II assessment of the *same* cohort, the number of 'at-risk' has increased (Afamasaga-Fuata'i et al. 2007a, b, 2008, 2010). On closer examination of content, SPELL I is purely computational items using the four operations ($+$, $\times$, $-$ and $\div$) whilst SPELL II has, in addition to this, word problems. Further research conducted by Afamasaga-Fuata'i et al.

(2007a, b, 2008, 2010), using a mathematics diagnostic test consisting of items mainly based on SMESC's Primary and Early Secondary Mathematics content, confirmed that the majority of Year 10 secondary students, from four local secondary schools that participated in the studies, also found that solving word problems is difficult. Most common difficulties were associated with the interpretation of word descriptions of both quantitative and mathematical relationships, the subsequent transformation of these interpretations into mathematical representations, the critical selection of relevant and appropriate strategies and procedures, and the synthesis of these to generate plausible solutions. Evidence showed that the more abstract the mathematical descriptions were, the more difficult it was for students to correctly answer the questions. Using the same diagnostic test, cohorts of post-secondary students enrolled in the Foundation programme at NUS also found that solving word problems is difficult. Empirical evidence from these foundation studies again verified the existence of difficulties and errors associated with solving word problems at a relatively lesser level compared to similar findings with the early secondary Year 10 students. These empirical studies and national results invariantly and consistently demonstrated students' difficulties solving word problems, as earlier identified by both SPELL tests, again at early secondary (at Year 10) and apparently persisting up to the end of secondary level many years later (Afamasaga-Fuata'i et al. 2007a, b, 2008, 2010).

In summary, the continuing negative trends of numeracy performances of national cohorts in their transition from Years 4 to 6 and given findings from Afamasaga-Fuata'i and her colleagues' secondary and foundation studies, sufficient empirical evidence exist to highlight an urgent need to innovatively and creatively reform the teaching of mathematics, at both primary and secondary levels, to align current practices, with more recent developments of educational learning theories such as those of the constructivist and sociocultural perspectives. The ultimate goal of such reforms is to motivate more students to consider mathematics as a potential area for further studies. Unquestionably therefore, a national need exists to offer innovative, constructivist and socioculturally driven learning programmes to specifically target the development of students' critical thinking, reasoning and analytical skills whilst engaged in working and communicating mathematically in their classrooms as they solve mathematics problems that are authentic, interesting and challenging. Whilst reforms in Samoan schools require the involvement of all stakeholders including SMESC and teachers, our overall study primarily focused on seeking empirical findings to inform further developments of innovative ideas and creative strategies for subsequent use in teacher education and primary classrooms.

Samoa's primary curriculum development project

Samoa's Ministry of Education, Sports and Culture is currently engaged in the last phases of a primary curriculum development project (ESP2) to develop new curricula in all subject areas including mathematics and trialling these through teacher workshops with the intention of introducing them in schools in 2013. To begin to address the empirically identified and confirmed problem-solving difficulties experienced by students at all levels (Afamasaga-Fuata'i et al. 2007a, b, 2008, 2010), the national mathematics curriculum writer (also the first author), in collaboration with the National Mathematics Subject Committee, designed and incorporated into the newly proposed primary mathematics curriculum, developmental (Piaget 1972) and working mathematically approaches, guided by the constructivist and sociocultural perspectives (Vygotsky 1978). Through encouraging students to engage with the five working mathematically processes (refer page 3) to solve mathematical problems in the classroom, students' critical thinking, reasoning and analytical skills are explicitly developed. The importance of these five sets of processes is captured in the form of a 'Working Mathematically' Strand with its own set of outcomes to be achieved by the end of each year from Years 1 to 8 following the models currently practiced in most international primary classrooms, including those in Australia (AAMT 2007), USA (NCTM 2007) and New Zealand (NZ Ministry of Education 2011). The content strands of the new mathematics curriculum are the Number and Operations, Patterns and Algebra, Data Analysis, Measurement, and Space and Geometry Strands complete with its own set of learning outcomes to be achieved by students at the end of each year level (SMESC 2013).

Philosophically and theoretically, the five 'working mathematically' processes are envisaged to be part of normal teaching, learning and assessment activities in mathematics classrooms. Whilst there are clear guidelines for the development of mathematical ideas and content over the 8 years of primary schooling, there is also the expectation that the 'working mathematically' processes are actively pursued and should underpin all learning experiences within the new Curriculum. To encourage the development and implementation of the 'working mathematically' processes in a classroom, a set of questions and strategies was provided to guide the PPTs as to the type of questions and type of strategies they should implement whenever students are engaged with solving a mathematical task.

Methodology

Strategy

Utilizing a single-group research design, the larger study monitored the collective impact of (a) the creative usage of authentic contexts as critical sites for mathematical investigations and (b) the innovative usage of mental computations and metacognitive tools to illustrate the conceptual structure of mathematics in a curriculum substrand and mathematics problems on students' mathematics attitudes and abilities to solve mathematics tasks. A pretest–post-test research strategy was used to enable attitudinal and mathematics competence data collection at the beginning and end of the study (Wiseman 1999). Furthermore, to explore PPTs developing understanding of mathematics and their proficiency in solving mathematical tasks, PPTs' responses to an open-ended item on the attitude questionnaire, interviews and student-constructed concept maps and vee diagrams including their reflections and investigation reports were collected.

Sample and duration

Purposive sampling targets a particular group of people. The identified group consisted of PPTs enrolled in the compulsory first-year mathematics methods (FYMM) course of their 2-year Diploma programme for the *second* time as their first attempt in the previous semester was unsuccessful. Also included in the course were prospective teachers enrolled in the Diploma (Early Childhood) and Diploma (Special Needs) programmes; typically, these students have relatively weaker mathematical backgrounds. The Diploma ECE and Special Needs PPTs had enrolled in the FYMM course as an elective, whereas it was compulsory for the General Diploma PPTs. Collectively, the three types of Diploma PPTs represented a unique group of students whose past and recent mathematical experiences and learning are not positive and would therefore be an ideal group to benefit from an infusion of innovative and creative approaches making mathematics learning more meaningful and relevant to their Samoan environment and daily practices. Furthermore, it was considered important that they be explicitly exposed to mathematical investigations of authentic contexts and working mathematically processes, mental computations and problem-solving strategies of the new Curriculum for which as graduate teachers, they would be expected to teach. There were eighty-four PPTs who participated in the study.

The course was for 14 weeks involving 10 weeks of face-to-face sessions meeting twice a week for 1-hour lectures and twice for 2-hourly small-group workshops. This was followed by a 3-week teaching practicum and a final week of reporting back to the university.

Data collection procedures

Mathematics diagnostic tests, course tests and assessment tasks

In the overall study, a diagnostic test of 38 items was administered two times (beginning and end) to track PPTs' developing numeracy and mathematical competence during the semester. The content covered by the test is based on SMESC's primary and early secondary mathematics curriculum. This diagnostic test and its various versions have been field-tested already with Samoan Foundation and Diploma students (see Afamasaga-Fuata'i 2011a; Afamasaga-Fuata'i et al. 2008, 2010).

Weekly lectures demonstrated the use of the innovative tools to illustrate mathematical ideas and interconnections, modelling the use of working mathematically processes, problem-solving strategies and examining everyday situations as authentic sources of mathematical ideas. During workshops, PPTs constructed concept maps to illustrate their conceptual understanding of key ideas based upon a topic area, mathematics problem and/or a set of learning outcomes. The PPTs also constructed vee diagrams to display both the conceptual bases of their methods (i.e. mathematical principles as justifications) and actual methods used to generate answers to the focus questions of their mathematical tasks.

Two course tests (Tests 1 and 2) and three assessment tasks (Task 1: *Authentic Mathematical Investigations*, Task 2: *Vee Diagrams and Concept Maps of Mathematics Problems* and Task 3: *Concept Map Analysis of Curriculum Substrands*) (see 'Appendix 1') examined the depth and breadth of PPTs' mathematical understanding, knowledge and skills of the new Curriculum as taught in the FYMM course and as examined through course tests, authentic mathematical investigations and applications of the innovative metacognitive tools.

Attitudinal questionnaires

To answer this article's specific focus questions, a pre-questionnaire was administered at the beginning of Week 1 of the semester with a post-questionnaire administered after PPTs' teaching practicum, in Week 14. The questionnaire had been used before by the first author with some Australian secondary students (see Afamasaga-Fuata'i 2011b) and with some Samoan foundation students (Afamasaga-Fuata'i and Lauano 2011).

Theoretically, items measuring students' mathematical attitudes were constructed based on a conceptualization of attitudes as comprising of *emotional*, *cognitive* and *behavioural* components. According to the educating and learning theories guiding this study, and the innovative and creative classroom practices that were implemented, PPTs were expected to be engaged in learning mathematics more meaningfully by thinking, reasoning and working mathematically to grasp the intended meaning of the activity, and having acknowledged that they have understood, deliberately choose to act (i.e. *acting*) on this understanding to determine and construct a possible solution. The items are therefore constructed to assess students' thinking about mathematics, their feelings towards the subject, its value and significance and including their actions when presented with a mathematical task. For example, whilst students are seeking to make sense of a learning activity, they are expected to be posing questions, strategically thinking and representing, reflecting and evaluating, and communicating mathematically. What they think about mathematics and what they perceive is the way to learn mathematics and solve problems are conceptualized to influence the types of actions they choose to take when undertaking a task.

Thirty-four (34) attitudinal items used a 5-point Likert scale with response categories ranging from Very Strongly Agree (VSA), Strongly Agree (SA), Neutral (N), Strongly Disagree (SD) to Very Strongly Disagree (VSD). Students' responses to the open question at the end of the questionnaire were collated and recorded in a spreadsheet. One item (Item 35: *I intend to continue taking mathematics next year*) used a 2-point rating scale: Yes or No. One open-ended item asked PPTs to provide an explanation of why they want to continue or not the study of mathematics following their current mathematics methods course. A copy of the questionnaire is in 'Appendix 2'.

End of semester interviews

Interviews with some PPTs were conducted during Week 14 of the semester and with some held in the first semester of the following year due to non-availability of PPTs. As interviews were voluntary and dependent on the availability of PPTs, only two interviews were possible directly after the semester with three others in the first week of the following semester. Interview questions were semistructured with additional probes to seek and clarify interviewees' views about the FYMM course, authentic investigations, metacognitive tools (vee diagrams and concept maps) and how the course could be further improved. PPTs' responses were tape-recorded and transcribed for analysis.

Data analysis

Quantitative analyses

Prospective teachers' responses to the 38 items in the diagnostic test were marked correct [1], incorrect [0] or blank [B]. Response categories of the attitudinal items in the questionnaire were coded from 1 to 5. For example, Very Strongly Agree was coded 5; Strongly Agree, coded 4; Neutral, coded 3; Strongly Disagree, coded 2; and Very Strongly Disagree, coded 1. The corresponding ratings from 5 down to 1 were reversed for negatively worded items and retained for positively worded items to reflect an overall increasingly positive attitude from 1 to 5.

Analyses of PPTs' responses to the attitudinal questionnaires and diagnostic tests (not included here) were conducted separately and independently using the Partial Credit Rasch Model and Dichotomous Rasch Model, respectively (Adams and Khoo 1996; Rasch 1980). The Rasch Measurement Model is an Item Response Theory Model which provides a robust approach to identifying an underlying latent trait or construct. In contrast to the test-level focus of classical test theory, the name *Item Response* (*IR*) *Theory* emphasizes its focus on the item, by modelling the *response* of an examinee of given ability to each *item* in the test (Bond and Fox 2001). The IR Theory is based on the idea that the probability of a correct/keyed response to an item is a mathematical logistic function of the difference between the person and item parameters. For example, the higher a person's ability estimate is relative to the difficulty of an item, the higher the probability the person gives a correct response on that item. When a person's location on the latent trait is equal to the difficulty of the item, there is by definition a 0.5 probability of a correct response in the Rasch model. The person parameter is called latent trait such as ability or agreeability; it may, for example, represent a person's intelligence or the strength of an attitude, respectively (Bond and Fox 2001). Whilst the IR Theory attempts to fit a model to observed data (Steinberg 2000), the Rasch model specifies requirements for fundamental measurement and thus requires adequate data-model fit before a test or research instrument can be claimed to measure a trait (Andrich 2004). Operationally, this means that the IR Theory approaches adjust model parameters to reflect the patterns observed in the data, whilst the Rasch approach requires that the data fit the Rasch model before claims regarding the presence of a latent trait can be considered valid. Therefore, under Rasch models, misfitting responses require diagnosis of the reason for the misfit and may be excluded from the data set if substantive explanations can be made that they do not address the latent trait (Smith 1990). Accordingly, evaluation of the fit of the data to the Rasch model provides information to determine the coherence of items to measure the underlying latent trait or theoretical construct [i.e. construct: mathematical competence (diagnostic test) and construct: mathematics attitude (questionnaire)].

The Rasch approach assumes guessing adds random noise to the data. As the noise is randomly distributed, provided sufficient items are tested, the rank-ordering of persons along the latent trait by raw score will not change, but will simply undergo a linear rescaling (Bond and Fox 2001; Rasch 1980; Item Response Theory 2012). A form of guessing correction is available within Rasch measurement by excluding all responses where person ability and item difficulty differ by preset amounts, so persons are not tested on items where guessing or unlucky mistakes are likely to affect results. However, if guessing is not random, arising through poorly written distractors that address an irrelevant trait, for example, then more sophisticated identification of pseudo-chance responses is needed to correct for guessing. Rasch fit statistics allow identification of unlikely responses that may be excluded from the analysis if they are attributed to guessing (Smith 1990).

The Rasch analysis results presented here provided estimates of PPTs' mathematics attitudes based on their endorsement of response categories of, or agreeability with, the questionnaire items. For the rating scale data from *Very Strongly Disagree* (coded 1) to *Very Strongly Agree* (coded 5) for each attitudinal item, not only does the analysis provide an item attitudinal estimate for each Likert stem, but it also provides a set of estimates for the five thresholds to mark the boundaries between the five Likert response categories. The item and its threshold estimates indicated the extent of 'difficulty to endorse' the attitudinal item stem. The Rasch model theorizes that each item reflects a different level of the latent trait being investigated and collectively the set of items are conceptualized and developed to reflect a developmental hierarchical trend from the 'easiest to endorse' or 'measures less of the latent trait' at the bottom towards the 'hardest to endorse' or 'measures much of the latent trait' towards the top of the logit continuum (Bond and Fox 2001).

The case estimates indicated the extent of PPTs' mathematical attitudes at the beginning and at the end of the study. That is, the person's attitudinal estimate 'represents the magnitude of *latent trait* of the individual, which is the human capacity or attribute measured by the test' (Lazarsfeld and Henry 1968). It might be a 'cognitive ability, physical ability, skill, knowledge, attitude, personality characteristic' (Item Response Theory 2012); in this study, it is students' mathematics attitudes.

A fundamental requirement of the Rasch model is the invariant comparison (Rasch 1980) between two persons irrespective of which set of items measuring the latent trait were used. To satisfy this requirement means that the researcher intentionally develops items that are valid for the purpose (Rasch Analysis 2005). Statistics from the Rasch analysis such as infit mean squares (ms) and standardized infit t are used to check the *person fit, item fit and overall data fit to the model,* and whether or not adding the scores is justified in the data. Data-model fit is therefore paramount and suggests that items are working together consistently to define an interpretable construct. According to Wright and Linacre (1994), a range of 0.6 to 1.4 infit ms values are considered an acceptable range to determine fit of items to the Rasch model for a survey that uses a rating scale. Based on analyses of numerous data sets, Linacre (in Wright and Linacre 1994) further noted that a range of ms fit statistics from 0.5 to 1.5 is productive for measurement purposes. For this article, the range of 0.5 to 1.5 will be considered an acceptable range to determine item fit to the model. The Rasch analysis also provides separation reliability indices to indicate how well the items and persons worked consistently to produce valid measures of the underlying variable.

If the data do fit the model adequately for the purpose of the questionnaire, then the Rasch analysis also linearizes the total score, which is bounded by 0 and the maximum score on the items, into measurements. Using the log of the odds of success, estimates are reported on a linear equal, interval scale (in logits–log odds units), which indicates the *location of the person* on the unidimensional continuum. This value is called a *parameter* in the model, and there can be only one number in a unidimensional framework. Unlike the classical test theory that simply asserts that the total score is the relevant statistic, with the Rasch model, the total score follows mathematically from the requirement of invariance of comparisons between persons and items (see Rasch analysis 2005; Bond and Fox 2001). If the purpose of the test or questionnaire is to assess a narrow range of skills and/or attitudes, respectively, then Rasch model and analysis may not be applicable (see 'Discussion' in Stacey and Steinle 2006).

Qualitative analyses of responses

Student-constructed concept maps and vee diagrams were marked according to analytic marking rubrics that were distributed in advance of each task's due date. Reflection components of Task 3 and other qualitative responses dispersed over Tasks 1–3 (not included here) were analysed using a grounded theory analysis approach to identify empirically grounded generalizations (categories and concepts within the data) and then based on these, to derive higher-level abstract generalizations about PPTs' actions and interactions with relatively novel ways of thinking and reasoning about, and problem-solving with, authentic contexts.

The response data from interviews and the open-ended questionnaire item were also analysed qualitatively to support the quantitative analysis, using a grounded theory analysis approach by identifying thematic categories to organize the presentation.

Results

Rasch analyses of cohort results

A total of 53 PPTs took the pre-questionnaire (preQ) whilst 45 took the post-questionnaire (posQ). Responses from both questionnaires ($n = 98$) were analysed using the Partial Credit Rasch Model as one data set to determine the relevant fit statistics and separation indices for the questionnaire as an instrument to measure PPTs' mathematics attitudes.

Fit of data to the model

Yu (2006) recommended that evaluation of person fit to the model should be done first before evaluating item fit to the model.

Person fit to the model Yu (2006, p. 23) and others (Bond and Fox 2001) define misfits among the cases (persons) as those who have an estimated attitudinal level that does not fit into the overall pattern as predicted by the Rasch model. Subsequently, after the initial analysis of responses from 98 cases, those with fit ms outside the acceptable range (0.5–1.50) were deleted ($n_{\text{cases}} = 19$). A second Rasch analysis with 79 cases ($\text{cases}_{\text{preQ}} = 43$ and $\text{cases}_{\text{posQ}} = 36$) and 34 items was conducted. With the item mean estimate theoretically set at 0.00, subsequent calibrations of person attitudinal estimates had a mean of 0.17 and standard deviation (SD) of 0.44. Also, the case mean infit ms value (1.01, SD = 0.64) was around the expected mean value of 1.00.

Item fit to the model An item mean infit ms value of 1.00 (also around the expected mean value of 1.00) was produced by the Rasch analysis using QUEST. Further inspection of individual items' infit ms values showed that all infit values were within the acceptable range (0.5–1.50).

Overall, the set of case and item infit ms statistics provided above both corroborate that the overall data fit the Rasch model.

Separation indices

Item and person separation indices are also part of the Rasch statistics generated from QUEST. Theoretically, if the objective is to measure a latent construct such as mathematics attitudes, then the focus is on how reliable the cases measured have been separated by the items in the questionnaire. From the analysis, the person separation index (i.e. 0.88) was closer to the ideal value of 1.00, suggesting the items worked together consistently and as a result the cases were reliably separated by the items in the questionnaire. The item separation index was closer to zero, indicating that whilst the cases were reliably separated by the items, the items were not reliably and sufficiently separated by the cases potentially due to its relatively small size ($n = 79$) and/or biased nature of the cohort in the sense that the sample consisted primarily of students who have not had very positive mathematical experiences in the recent past and were only taking the course as a requirement for their Diploma programme or as an elective upon the strong recommendation of their programme coordinator for the Diploma-Special Needs and Diploma-Early Childhood students. For the purpose of the attitudinal data presented here, it is acknowledged again that the study's cohort uniquely consisted of PPTs whose mathematics attitudes would be generally around average to low. Evidence to further support this point is provided in the next section.

Pre-questionnaire and post-questionnaire variable maps

The QUEST item-person for the pre- and post-questionnaires is shown in Figs. 1 and 2. The two maps displayed item threshold estimates, denoted by item number and response category (i.e. 18.5 in Fig. 1), on the right of the vertical logit continuum, towards the top are the hardest-to-endorse item response categories denoting very positive attitudes, whilst those relatively easiest-to-endorse item response categories describing negative attitudes are towards the bottom. To the left of the same vertical logit scale is the distribution of cases, represented by *X*, indicating the location of persons' attitudinal estimates from those with positive attitudes towards the top and those with relatively negative attitudinal estimates towards the bottom. Similarly, the item threshold estimates are spread out hierarchically from the top towards the bottom of the continuum.

Findings (adequate fit of the data to the model and high person separation index) collectively indicated that the persons who attempted the items performed in expected ways. That is, in both Figs. 1 and 2, those with very positive attitudes were separated out along the continuum towards the top and those with very negative attitudes towards the bottom.

In the pre-questionnaire (Fig. 1) item-person (or variable) map, students with the top three highest attitudinal estimates were located at 1.41, 1.41 and 1.21 logits, respectively, as indicated by the top three *X*s on the right of the vertical lines where each student is represented by an *X*. The four lowest attitudinal estimates are indicated by the last four *X*s at −0.51, −0.57, −0.71 and −1.00 logits, respectively. In between these highest and lowest estimates is the distribution of the rest of the estimates in decreasing order of positive attitudes.

Item threshold estimates, as represented by item number and response category on the right of the vertical lines, are located according to how much of the latent trait they are purported to measure. That is, in both Figs. 1 and 2, item threshold estimates located towards the top end of the logit continuum have high attitudinal estimates and are hardest-to-endorse response categories reflecting increasingly positive attitudes, whilst those towards the bottom have the least positive attitudinal estimates and are easiest-to-endorse response categories, thus more reflective of increasingly negative attitudes. For example, the top three item thresholds, namely 18.5, 10.5 and 4.5, indicated the most positive response category coded 5 for: Very Strongly Disagree, as responses to the negatively worded Item 18 (*I feel nervous when doing mathematics*), Item 10 (*I get worried when solving a problem that is different from the ones done in class*) and Item 4 (*Mathematics makes me feel uncomfortable and impatient*). These three item thresholds had attitudinal estimates of 2.73, 2.09 and 1.91 logits, respectively. Item analysis data also showed that these response categories (rated 5) were endorsed by 2.3 % (1/43) for Items 18 and 10 with 4.7 % (2/43) for Item 4. It should be noted also that item threshold estimates that are located in a horizontal row indicate estimates that are in close proximity to each other given the constraints of the scale divisions on the logit continuum.

During the Rasch analysis, the mean item estimate is theoretically set to 0.00 logits before the case estimates are calibrated. Accordingly, QUEST-generated statistics showed a comparatively higher case estimate mean at 0.31 logits (SD = 0.47) relative to the mean item estimate of 0.00. This higher case estimate mean suggested that, on average, the cohort appeared relatively more positive as a group. In the posQ, the mean case estimate was lower at −0.01 (SD = 0.44) logits, implying a drop in the cohort's attitudinal mean. Summary case statistics from the preQ and posQ responses are provided below in Table 1.

```
----------------------------------------------------------------------
Item Estimates (Thresholds)preq on all (N = 43 L = 34 Prob Level=0.50)
----------------------------------------------------------------------
  3.0                                       |
                                            |
                                            |     18.5
                                            |
                                            |
                                            |
                                            |     10.5
                                            |
                                            |
                                            |      4.5
                                            |
                                            |     11.5
  2.0                                       |      6.5   12.4
                                            |     33.5
          +3 SD                             |      9.5
                                            |
                                            |      1.5    3.5
                                            |     10.4   16.5
        Highly Postive                      |     12.3   23.5
                                       XX   |     14.4
                                        X   |      2.4   19.5
          +2 SD                             |     15.5
                                        X   |      4.4    5.5    8.5   18.4
          Very Positive                     |     13.5
  1.0                                   X   |      7.4   22.5
          +1 SD                         X   |      6.4   14.3   30.5
                                        X   |     25.5   34.5
                                        X   |      2.3    9.4   23.4   29.5
          Positive                     XX   |     17.5   33.4
                                     XXXX   |     10.3   12.2   18.3   24.5
Case Mean 0.31                      XXXXX   |     16.4   19.4   21.5   32.5
                                        X   |      4.3    7.3   22.4   31.5
                                      XXX   |     20.5   28.5
                                     XXXX   |      6.3    8.4   33.3
          Negative                   XXXX   |      9.3   19.3
  0.0                                 XXX   |      1.4   21.4   23.3   27.5        Item Mean 0.00
          -1 SD                         X   |     16.3   22.3
                                       XX   |     11.4   29.4
                                       XX   |      3.4   14.2
                                            |      5.4    8.3   11.3   13.4   17.4
          Very Negative                     |     15.4   21.3   26.5
          -2 SD                         X   |      7.2   20.4   25.4   31.4
                                        X   |      6.2   27.4   28.4   32.4
                                        X   |
        Highly Negative                     |      4.2   17.3   24.3
                                            |     10.2   15.3   18.2
                                            |     19.2   20.3   31.3
 -1.0     -3 SD                         X   |     11.2   13.3   16.2   26.4   30.3
                                            |      3.3   25.3
                                            |      1.3
                                            |     17.2   21.2   22.2   28.3   29.3
                                            |     23.2
                                            |      2.2
                                            |      5.3   27.3
                                            |     25.2   32.2
                                            |      9.2   20.2   31.2   34.4
                                            |      8.2   13.2   26.3
                                            |
                                            |
 -2.0                                       |
                                            |
                                            |     33.2
                                            |
                                            |
                                            |
                                            |
                                            |
                                            |
                                            |
                                            |
                                            |
 -3.0                                       |
--------------------------------------------------------------------------------------
  Each X represents    1 students   Some thresholds could not be fitted to the display
======================================================================================
```

Fig. 1 Pre-questionnaire variable map of person-item threshold estimates

```
---------------------------------------------------------------------------
Item Estimates (Thresholds) posq on all (N = 36 L = 34 Prob Level=0.50)
---------------------------------------------------------------------------
  3.0                               |
                                    |
                                    |
                                    |
                                    |
                                    |
                                    |      14.5
                                    |
                                    |
                                    |
                                    |
                                    |      19.5
  2.0                               |
                                    |
                                    |
+3 SD   ____________________________|
                                    |       5.5
        Highly Positive             |       2.5   25.5
                                    |       9.5   23.5   29.5
                                    |       7.5   18.5   30.5   32.5   33.5
+2 SD   ____________________________|      13.5   15.5
                                    |       4.5   20.5   22.5
                                    |       6.5    8.5
  1.0   Very Positive               |       3.5   10.5
                                    |      24.5
                                    |       1.5   11.5   12.5   21.5
+1 SD   ____________________________|      17.5   27.5
                                  X |       9.4   19.4   34.5
        Positive                 XX |      12.4   16.5
                                  X |       7.4    8.4   11.4   18.4   23.4
                                 XX |      10.4   12.3   14.4   19.3   28.5
                                  X |       5.4    9.3
PreQ Case Mean 0.31 ________________|       1.4    2.4   16.4   33.4
                               XXXX |       6.4   11.3   22.4   31.5
                            XXXXXXX |       3.4    4.4   14.3
        Negative                  X |       8.3   16.3   18.3   26.5
  0.0                          XXXX |       2.3    7.3   13.4   15.4   17.4   Item Mean -0.01
                                 XX |_______1.3___10.3___12.2___30.4____________
-1 SD   _________________________XX_|       4.3    6.3   21.4   22.3
                                 XX |      15.3   20.4   27.4   28.4   29.4
        Very Negative           XXX |       3.3    9.2   14.2   17.3   19.2
                                  X |       5.3   13.3   24.3
                                    |      21.3   31.4   32.3   34.4
-2 SD   __________________________X_|      27.3   33.3
                                    |       1.2    8.2   20.3   28.3   29.3
                                    |       2.2   25.3   26.3   26.4   30.3
                                    |      10.2   17.2   20.2   27.2
                                  X |       5.2   11.2   13.2   18.2   22.2
 -1.0                               |       3.2   25.2   29.2   30.2
        Highly Negative             |      16.2   23.2
                                    |       6.2   32.2
                                    |      28.2
                                    |       7.2   15.2   21.2   26.2
                                    |       4.2
-3 SD   __________________________X_|
                                    |      34.2
                                    |
                                    |
                                    |
                                    |      33.2
 -2.0                               |
                                    |
                                    |
                                    |
                                    |
                                    |
                                    |
                                    |
                                    |
                                    |
                                    |
 -3.0                               |
-------------------------------------------------------------------------------
  Each X represents 1 student. Some thresholds could not be fitted to the display
===============================================================================
```

Fig. 2 Post-questionnaire variable map of person-item threshold estimates

Table 1 Pre-questionnaire and post-questionnaire case estimate results

Pre-questionnaire		Post-questionnaire	
Summary of case estimates ($N = 43$)		Summary of case estimates ($N = 36$)	
Mean	0.31	Mean	−0.01
SD	0.50	SD	0.44
SD (adjusted)	0.47	SD (adjusted)	0.41
Reliability of estimate	0.87	Reliability of estimate	0.88
Infit ms		Infit ms	
Mean	1.01	Mean	1.04
SD	0.40	SD	0.69
Outfit ms mean		Outfit ms mean	
Mean	1.01	Mean	1.06
SD	0.38	SD	0.77

With the case distributions in the preQ variable map (Fig. 1), about three persons were at the top around 1.30 logits compared to a top clump of about seven students at lower values around about 0.7 logits in the posQ variable map (Fig. 2). Figure 1 shows a narrow person distribution around its mean with person estimates located within the range of ±1 logit around zero logit suggesting a relatively narrower range of attitudinal levels compared to the much broader variation of item threshold estimates from around −2 logits up to just under +3 logits around the item mean of 0.00 logits. Furthermore, the existence of 'tails' at the extreme ends of the distribution of item threshold estimates implied that more cases with attitudinal estimates at the highest and lowest ends would be required to further improve the item separation index signalled earlier.

One way of interpreting more meaningfully the results in Fig. 1 is by grouping attitudinal estimates in terms of standard deviations from the mean and then labelling these groups to facilitate discussion. For example, estimates located at 'at least 2 SD from the mean' may be labelled as representing '*highly positive*' attitudes, 'at least 1 SD but less than 2 SD from the mean' as '*very positive*' attitudes, 'no more than 1 SD and greater than or equal to the mean' as '*positive*' attitudes, 'less than the mean and no more than 1 SD below the mean' as '*negative*' attitudes, 'greater than 1 SD and less than 2 SD below the mean' as '*very negative*' attitudes and 'more than 2 SD below the mean' as '*highly negative*' attitudes. In so doing, Fig. 1 shows that only three PPTs may be classified as having highly positive attitudes, four in the very positive category with four and two in the very negative and highly negative categories, respectively, with the rest classified as having either positive or negative attitudes. It is also acknowledged that border cases (near the category boundaries) could easily be in the adjacent category given their respective standard errors.

With the posQ variable map (Fig. 2), the top two item threshold estimates (i.e. response category 5) indicated strong disagreement about forgetting mathematics concepts learnt in previous classes (Item 14: *I have forgotten many of the mathematical concepts that I have learnt in previous mathematics classes*) and strong agreement with Item 19: *My most favourite subject is mathematics*. These two item threshold estimates were located at 2.53 and 2.13 logits, respectively. Item analysis data showed that for each item stem, only 2.8 % (1/36) endorsed this response category.

A comparison of the preQ and posQ person-item maps and Table 1 statistics suggested that the lower posQ mean case estimate was indicating a deterioration (negative improvement) in the cohort's mathematics attitudes with a moderate to large Cohen's d value of -0.73, demonstrating a significant practical difference. This d value means that a practically significant effect attitudinally (Cohen 1977) did occur by the end of the semester in terms of some collective impact of PPTs' learning experiences, which included as well the usage of innovative metacognitive tools and conduct of authentic investigations. The deterioration in the cohort's attitudes by the end of the study seems to suggest that whilst their learning experiences and exposure to a variety of innovative approaches may have had some impact on their learning of, and perceptions about, mathematics, it may also have resulted in a self-realization that there is much that they needed to learn more effectively in order to become competent and confident future teachers of mathematics (see interview responses presented later).

The large practical difference may be further understood by an examination of changes from the preQ to posQ, in proportions of students within each of the six attitudinal categories (from Highly Positive through to Highly Negative) using the same category boundaries (i.e. mean $\pm$ SDs) from the preQ. Tabulated in Table 2 are the number and percentage per attitudinal category whilst graphed in Fig. 3 are the proportions per category for comparison purposes. Clearly, Fig. 3 illustrates that by the posQ, no more PPTs remained in the top two attitudinal categories (Highly Positive and Very Positive) with a reduction of about 11 % in the Positive category whilst corresponding increases are noted in the next two categories (Negative and Very Negative) with a slight increase in the Highly Negative category. If the categories are collapsed to form two more general ones, namely Positive and Negative attitudes, then by the posQ, only 19.4 % PPTs indicated Positive attitudes with the rest (80.6 %) indicating Negative attitudes compared to 46.5 and 53.5 %, respectively, in the preQ. Both the mean estimates statistics (Table 1) and proportions by attitudinal category (Table 2) further corroborated that the post-questionnaire cohort appeared, on average, to have relatively lower attitudes towards mathematics. Further examination of this negative trend in cohort attitudes is provided next using item variations. A more detailed item-by-item comparison was considered useful in order to provide a more nuanced understanding of PPTs' negative attitudinal change in terms of their thinking about, and feelings about and towards, mathematics and mathematics learning, and the kinds of actions they resort to when learning mathematics meaningfully or not and/or asked to complete a mathematical task particularly given the nature of the innovative approaches, strategies and tools introduced in the course.

A QUEST-generated comparison of preQ to posQ item variations showed that overall (Chi SQ $= 85.25$, $df = 18$, $p = 0.00$) there was a highly significant difference between item estimates in terms of standardized differences (see Table 3). To provide some useful connections or not to the theoretical underpinnings of the study, signed standardized difference values were examined further to identify and then examine more closely the types of items that became more positive (negative $d1$–$d2$ standardized values) and those that became more negative (positive $d1$–$d2$ standardized values) by the post-questionnaire.

To elaborate further, the 'more positive' items indicated those whose posQ estimates and in turn their locations on the logit continuum have shifted higher than their preQ locations. In contrast, the 'more negative' items were those whose posQ estimates have decreased, and in turn their locations on the continuum were lower than their preQ values. Accordingly, out of the 19 items compared in Table 3, eleven (11) items became 'more positive' (negative $d1$–$d2$ standardized values) whereas eight (8) became 'more negative' (positive $d1$–$d2$ standardized values). Provided in Table 4 is a list of these items by

Table 2 Number and percentage of students in attitudinal categories

Attitudinal categories	PreQ Number	PreQ %	PosQ Number	PosQ %
Number of students	43		36	
Highly positive	3	7.0	0	0
Very positive	4	9.3	0	0
Positive	13	30.2	7	19.4
Negative	17	39.5	18	50.0
Very negative	4	9.3	9	25.0
Highly negative	2	4.7	2	5.6

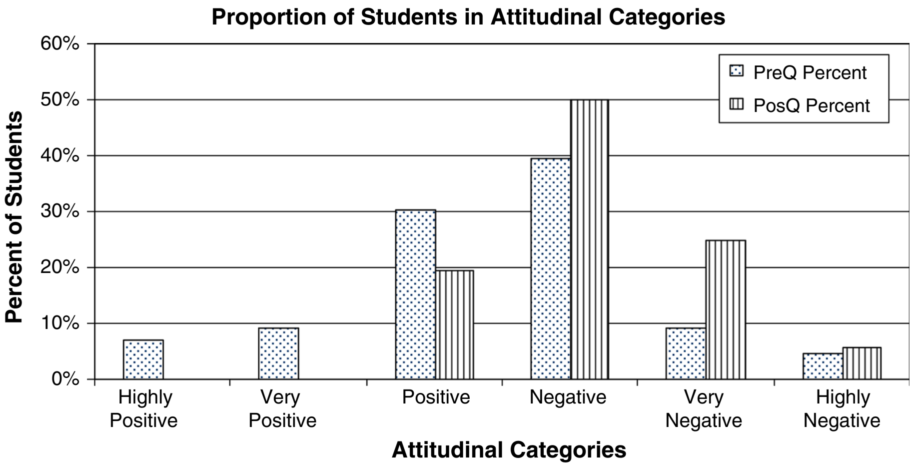

Fig. 3 Pre-questionnaire variable map of person-item threshold estimates

category. Of the eleven items that became 'more positive' (indicating higher posQ estimates than preQ values), five have standardized differences that were significant at the 5 % level. These items indicated PPTs' agreement with the usefulness or utilitarian value and relevance of mathematics skills to daily living (Item 20) and application in other subject areas (Item 32) including their belief that they can solve any mathematics problem by using their existing knowledge (Item 13).

Also significant were the items about mathematics being their favourite subject (Item 19) and expressing their liking for, and enjoyment in studying mathematics (Item 8). The rest of the 'more positive' items included those about the value of models or diagrams to clarify their thinking when solving problems (Item 25), their willingness to improve their understanding of mathematics (Item 31), their firm belief in their ability to cope with new problems (Item 9) and the great sense of satisfaction (like winning a game) when they are successful in solving a problem on their own (Item 17).

With the eight items that became more negative (indicating lower posQ estimates than their preQ values), the four highly significant ($p = 0.00$) ones described feelings of discomfort and impatience with mathematics (Item 4), getting worried when solving a problem that is different from the ones done in class (Item 10), and nervousness when

Table 3 Comparison of pre- and post-questionnaire item estimates and standardized differences

	Delta		Adjusted delta		Difference		χ^2	p
	Preq SE	Posq SE	Preq $d1$	Posq $d2$	$d1$–$d2$	$d1$–$d2$ standardized		
Item 4	0.73	−0.07	0.70	−0.10	0.70	3.79	14.34	0.00**
	0.15	0.14						
Item 6	0.63	−0.01	0.60	−0.04	0.64	3.15	9.92	0.00**
	0.14	0.14						
Item 8	−0.18	0.25	−0.21	0.21	−0.42	−2.08	4.31	0.04*
	0.14	0.14						
Item 9	0.24	0.55	0.21	0.51	−0.30	−1.42	2.01	0.16
	0.15	0.15						
Item 10	0.94	0.14	0.91	0.07	0.84	3.76	14.13	0.00**
	0.17	0.10						
Item 11	0.16	0.17	0.13	0.14	−0.01	−0.03	0.00	0.98
	0.15	0.14						
Item 13	−0.48	−0.04	−0.51	−0.07	−0.44	−1.93	3.71	0.03*
	0.17	0.15						
Item 16	0.24	−0.06	0.21	−0.09	0.30	1.57	2.38	0.12
	0.13	0.13						
Item 17	−0.41	−0.11	−0.44	−0.14	−0.30	−1.44	2.45	0.12
	0.16	0.14						
Item 18	0.92	0.25	0.88	0.22	0.67	3.05	9.31	0.00**
	0.16	0.15						
Item 19	0.26	0.72	0.23	0.69	−0.46	−2.24	5.01	0.03*
	0.13	0.16						
Item 20	−0.67	−0.11	−0.70	−0.15	−0.55	−2.46	6.04	0.01*
	0.17	0.15						
Item 21	−0.27	−0.32	−0.30	−0.35	0.05	0.24	0.06	0.81
	0.14	0.15						
Item 22	0.01	0.04	−0.02	0.01	−0.03	−0.16	0.02	0.88
	0.13	0.14						
Item 23	0.21	0.20	0.17	0.16	0.01	0.06	0.00	0.95
	0.14	0.15						
Item 25	−0.57	−0.14	−0.60	−0.18	−0.42	−1.75	3.06	0.08
	0.18	0.17						
Item 31	−0.65	−0.46	−0.68	−0.49	−0.19	−0.85	0.73	0.39
	0.17	0.15						
Item 32	−0.65	−0.10	−0.68	−0.14	−0.55	−2.36	5.56	0.02*
	0.18	0.15						
Item 33	0.14	−0.21	0.11	−0.25	0.36	1.58	2.51	0.11
	0.14	0.17						
Means	0.04	0.04	0.00	0.00	$\chi^2 = 85.25$		($df = 18$, $p = 0.00$)	

SE standard error

** Highly significant; * significant at the 5 % level

Table 4 'More positive' and 'more negative' questionnaire items

Standardized differences	Items that became more positive (positive item variations)	Standardized differences	Items that became more negative (negative item variations)
−2.46*	**Item 20**: Mathematics classes provide the opportunity to learn skills that are useful in daily living	3.79**	**Item 4**: Mathematics makes me feel uncomfortable and impatient
−2.36*	**Item 32**: The skills I learn in mathematics will help me in other subjects at school	3.76**	**Item 10**: I get worried when solving a problem that is different from the ones done in class
−2.24*	**Item 19**: My most favourite subject is mathematics	3.15**	**Item 6**: I am nervous in mathematics classes because I feel I cannot do mathematics
−2.08*	**Item 8**: I really like mathematics; it's enjoyable	3.05**	**Item 18**: I feel nervous when doing mathematics
−1.93*	**Item 13**: I believe that if I use what I know already, I can solve any mathematics problem	1.58	**Item 33**: I do not have to understand mathematics, I simply memorize the steps to solve a problem
−1.75	**Item 25**: I learn to think more clearly in mathematics if I make a model or draw diagrams of the problem.	1.57	**Item 16**: If I cannot solve a mathematics problem, I just ignore it
−1.44	**Item 17**: Successfully solving a problem on my own provides satisfaction similar to winning a game	0.24	**Item 21**: To succeed in school, you don't need to be good in mathematics
−1.42	**Item 9**: I can cope with a new problem because I am good in mathematics	0.06	**Item 23**: I don't think I could learn advanced mathematics, even if I really tried
−0.85	**Item 31**: I am interested and willing to improve my understanding of mathematics		

** Highly significant ($p = 0.00$); * significant at the 5 % level doing mathematics or feel they cannot do mathematics (Items 6 and 18). The other items described PPTs' belief that given a problem, they do not have to understand mathematics; instead, they simply memorize the steps to solve it (Item 33) or just ignore it (Item 16). The other two items indicated that they do not need to be good in mathematics to be successful in school (Item 21) and their feeling of helplessness in learning advanced mathematics even if they tried (Item 23).

Overall, it appeared that the top items demonstrating positive estimate variations described PPTs' strong feelings of liking, interest and enjoyment; strong perception and/or awareness of the utilitarian value and relevance of mathematics skills in daily living and cross-curricular applications; strong beliefs in their ability and capacity to use their existing knowledge to solve any problem including new ones; and their insight about the value of models or diagrams to clarify one's thinking. The sense of great accomplishment after independently solving a problem appeared to positively influence their attitudes towards mathematics. In contrast, items with negative variations by the end of the semester demonstrated entrenched negative feelings of discomfort, impatience, anxiety and nervousness. Further suggested was PPTs' strong agreement with memorization strategies when solving problems they do not understand, reflecting their existing beliefs and perceptions about the nature of mathematics. At worst if they cannot solve a problem, they strongly agreed that they would ignore it, suggesting a lack of motivation to persist and/or try. Also evident was PPTs' agreement that they can be successful in school without being good in mathematics and that even if they tried, they would be unable to learn advanced mathematics, suggesting a sense of acceptance of their perceived mathematical ability and capacity for further studies.

Case study: analyses of paired attitudinal estimates

The authors were also interested in a case study of those PPTs that completed both the pre- and post-questionnaires. Provided in Table 5 below are these PPTs' ($n = 17$) preQ and posQ attitudinal estimates, including whether or not they passed the FYMM course, their final course mark and whether or not he or she was a repeater from the previous semester or first-time taker from the Diploma-Early Childhood and Diploma-Special Needs programmes. Of the seventeen (17) PPTs, only one was a first-time taker, the rest were all repeaters. From the sixteen (16) repeaters, seven (44 %) passed the course. In the last column are the attitudinal changes defined as posQ–preQ estimates in which positive attitudinal changes reflected higher posQ values than preQ values.

A means comparison of the paired estimates produced a moderate Cohen's d (-0.33), which indicated a moderate relationship between the paired sets of estimates (preQ and posQ) for the seventeen PPTs that answered both questionnaires. This moderate value implies that a negative practical effect attitudinally (Cohen 1977) did take place by the end of the semester, in terms of any collective impact of PPTs' learning experiences, which included the usage of innovative metacognitive tools and conduct of authentic investigations. Graphed in Fig. 4 are the 17 attitudinal changes to indicate their relative amounts. Of the 53 % (9/17) that had positive attitudinal changes, 56 % (5/9) passed the course and 44 % (4/9) failed.

Visually displayed (Fig. 4) are the relatively smaller amounts of positive changes compared to the relatively bigger negative changes. The majority (63 %, 9/17) of the seventeen PPTs had positive attitudinal changes with 47 % (8/17) having negative attitudinal changes. Illustrated in Fig. 5 are the actual preQ and posQ values for positive changes (Fig. 5a) and for negative changes (Fig. 5b) relative to the paired group mean estimates. The positive changes

Table 5 Paired attitudinal measures and attitudinal changes

Student ID	R/1st	Final mark	Pass/fail	PreQ	PosQ	PosQ–preQ
1	R	17	Fail	−0.94	−0.49	0.45
2	R	40	Fail	−0.54	−0.25	0.29
3	R	53	Pass	0.33	0.61	0.28
4	R	36	Fail	−0.16	0.09	0.25
5	1st	78	Pass	−0.13	0.05	0.18
6	R	56	Pass	0.02	0.15	0.13
7	R	61	Pass	−0.48	−0.36	0.12
8	R	23	Fail	−0.01	0.09	0.1
9	R	50	Pass	0.14	0.22	0.08
10	R	38	Fail	−0.19	−0.21	−0.02
11	R	34	Fail	0.59	0.53	−0.06
12	R	61	Pass	0.56	0.45	−0.11
13	R	58	Pass	0.36	0.09	−0.27
14	R	50	Pass	0.11	−0.33	−0.44
15	R	43	Fail	−0.67	−1.51	−0.84
16	R	26	Fail	1.35	0.24	−1.11
17	R	24	Fail	1.35	−0.96	−2.31
Mean		44		0.1	−0.09	−0.19
SD		16.47		0.63	0.54	0.68

1st first time

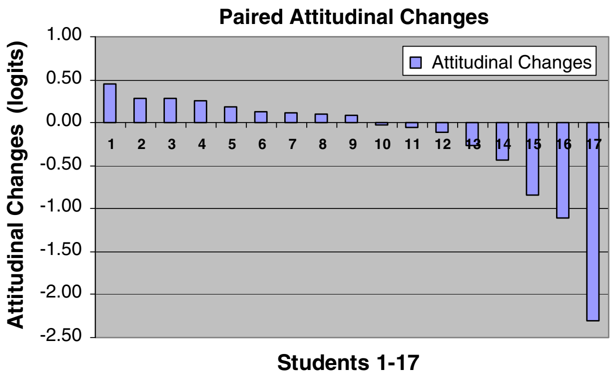

Fig. 4 Paired attitudinal changes

were noteworthy. Of the nine students with positive attitudinal changes, 67 % (6/9) of them had posQ higher than the paired group mean (-0.09 logits, $n = 17$, see Table 5). It is also pleasing to note that Students 1 and 2, initially with negative attitudes (below −0.50 logits), became more positive (above −0.50 logits) by at least 0.30 logits (see Figs. 4, 5a) by the end of the semester. The majority of PPTs with positive attitudinal changes (56 %, 5/9) passed the course. With negative attitudinal changes (see Figs. 4, 5b), the majority (62.5 %, 5/8) were less than one-half logit with 37.5 % (3/8), indicating around one or at least one logit change. Of the latter three students (Students 15, 16 and 17), initial attitudinal estimate of Student 15

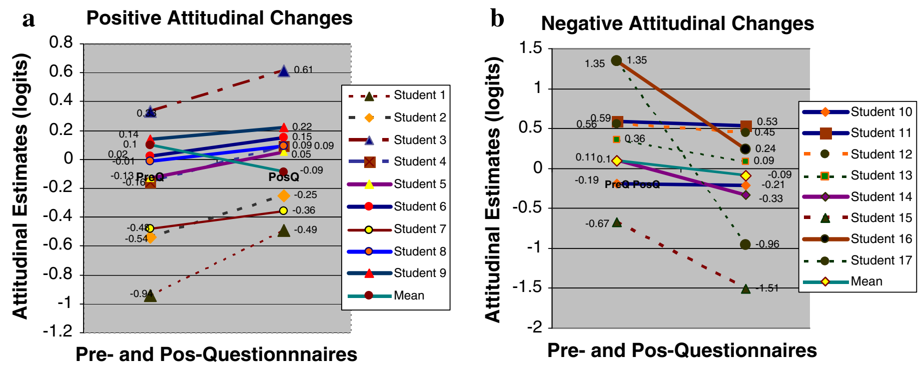

Fig. 5 Positive and negative attitudinal changes

(−0.67 logits) was in the Very Negative category, and after the semester, it shifted to the Highly Negative category (−1.51 logits). This pattern of impact reflects that of initial negative attitudes becoming further entrenched as a result of the course. The case of Student 17 demonstrates that of shifting from a Highly Positive attitude to a Highly Negative one. In comparison, the case of Student 16 demonstrates that of a Highly Positive attitude becoming Negative by the end of the course. The majority of PPTs with negative attitudinal changes (63 %, 5/8) failed the course.

Analyses of interview responses and item 35 open responses

PPTs' responses (Yes or No) to Item 35 (*I intend to continue taking mathematics next year*) were counted and are provided in Table 6. Evidently, the number of PPTs intending to continue taking mathematics is in the majority from both questionnaires (64 % preQ and 73 % posQ) with an increased percentage intending to take another mathematics course in the following year by the end of the semester. Part of the reason may be because this FYMM course is the first of two compulsory mathematics methods course for the general Diploma PPTs. What is interesting is the extra 9 % of the PPTs who deliberately opted to continue their mathematics studies.

As described earlier, there are two types of PPTs in the study, the general Diploma PPTs who are required to take a second compulsory course after this FYMM course and the others (Diploma-Early Childhood and Diploma-Special Needs) who do not have to take another mathematics (methods) course. PPTs' explanations in the post-questionnaire (Item 35) and interview responses were qualitatively analysed to identify PPTs' main views and perceptions at the end of the semester after all the required course assessments and their teaching practicum to shed some light on the observed attitudinal change. From the responses of the 73 % of PPTs that answered 'Yes' to Item 35 and the interviewees, four main themes were identified. These included their (1) motivation to take another mathematics course, (2) perceived need to improve mathematics, (3) personal perceptions of primary mathematics and (4) course feedback. These themes are briefly described next.

I am motivated to take mathematics

Many students said they would continue taking mathematics the following year for a variety of reasons but primarily because they want to do well in all their general Diploma

Table 6 Percentage of Item 5 responses (yes or no) from the pre- and post-questionnaires

	Yes (%)	No (%)
Pre-questionnaire ($N = 53$)	34 (64 %)	19 (36 %)
Post-questionnaire ($N = 45$)	32 (73 %)	11 (27 %)

papers and to graduate. Some wanted to do more mathematics courses to increase their interest in the subject (e.g. SY18) and to improve their chances of getting a good job (e.g. SY31).

I need to improve my mathematics

Many PPTs wanted to continue taking another mathematics course because they wanted to improve their mathematics knowledge and skills not only for themselves but also for the sake of their future students (e.g. SY20 and SY28), and as one Diploma-Early Childhood student said, she wanted 'to teach the students the right methods used and given to them'. Many PPTs also acknowledged that it was their responsibility to continue taking and passing their required mathematics courses as they will have to 'teach all the subjects including maths so every subject is compulsory' and that they should be 'excellent in every area required for a responsible teacher to teach at any school' (e.g. SY24).

Personal perceptions of the FYMM course and primary mathematics

All five interviewees were positive about their learning experiences in the FYMM course, and as such, their perceptions were either reinforced if positive, or changed if they were initially negative. The two PPTs interviewed at the end of the semester had in fact tried to withdraw within the first 2 weeks of the semester, saying that the course was too hard and it was only an option for them. They said: 'it wasn't worth all this '*faatigaulu*' or "headaches"' (referring to the thinking and reasoning requested of them as they applied the working mathematically processes, mental computations and various multiple problem-solving strategies). However, with some encouragement from the second author, the two PPTs stayed the course and at the end of the semester, they commented: '(we) think the whole course was excellent once (we) understood' and 'this course can wake me up'. Other interviewees commented: 'the course (made us feel) excited about teaching mathematics in (our) class(es)'. Some PPTs commented that unless they understood, they would be negative not only about the whole course, but the subject as well; for example, she said: 'my friend did not understand (any part of) the course and does not like math at all, he wished this course was not compulsory'.

Some PPTs compared their mathematical experiences in the FYMM to the type of mathematics they found was being taught in schools during their teaching practicum. They perceived that the nature and level of mathematics taught in the FYMM course should be the same as that taught in primary level; they said 'a lot of things that they teach is not really relevant to what we can teach at primary' (e.g. SY3, SY14) and 'there were lots of new methods that we should not apply when teaching primary students' (SY32).

Course feedback

SN1, a Diploma-Early Childhood PPT who passed the FYMM course and who was initially quite reluctant to take the FYMM course, explained: 'I might continue if I have the chance but I am moving off island so I'm not sure what the future holds for me but I really enjoyed what I've learnt and must admit I have learnt something important out of it. Thanks'. Also in the interview, SN1 explained that '(she knew) exactly the method(s) and ideas in teaching mathematics (at) each level'. Regarding the metacognitive tools of concept mapping and vee diagrams, SN1 continued to say '…I love concept mapping and vee-diagrams …its like a plan of what you are going to do'. The importance of mathematics being a metacognitive activity was further illustrated by SY32, another Diploma-Early Childhood PPT, who said that 'for me it was good in a way that (it) makes my brain work'.

The five interviewees all commented that the course was important and useful. For example, they said: '(w)e learnt a lot of new methods that were given … we used them with our own children'. In terms of mental strategies learnt and applied, this was one area that was very clearly stressed by all participants throughout their interviews as being a key outcome of the course.

Discussions and main findings

In addition to the usual course assessment tasks of course tests and written assignments, the PPTs applied innovative metacognitive tools to working mathematically processes, mental computations and problem-solving strategies whilst conducting their authentic mathematical investigations. Through pre- and post-questionnaires, including an open-ended item and end of study interviews, subsequent PPTs' responses were Rasch and qualitatively analysed to provide answers to the two focus questions: (1) What were the primary prospective teachers' general attitudes to mathematics at the *beginning* and *end* of the one-semester mathematics methods course? (2) What factors appeared most influential in changing Primary prospective teachers' attitudes towards mathematics? Discussion of the results is framed around these two main questions.

The quantitative evidence from the posQ variable map showed that the two highest item estimates were those describing the importance of remembering concepts learnt in previous classes and mathematics being their most favourite subject. These statements suggested plausible connections to the fundamental ideas underpinning the construction of the two metacognitive tools (concept maps and vee diagrams) and the investigation of authentic contexts for the manifestations and applications of mathematical ideas (whose relevant empirical data are not included in this article). Through the (successful) application of working mathematically processes, mental computations and multiple problem-solving strategies, with or without the construction of concept maps and vee diagrams, PPTs were supported and scaffolded as they learnt to strategically identify and meaningfully understand and appreciate mathematical ideas, their interconnections and various applications in selecting appropriate methods in solving mathematical tasks or conducting investigations. According to Schoenfeld (1991) and Ernest (1999), routinely incorporating such processes and innovative strategies to change classroom practices can actually shape the behaviour and understanding of students by making it more natural for them to think and reason mathematically. When successful, the potential of nurturing positive attitudes towards mathematics and problem-solving in general is powerful.

However, if unsuccessful, then negative attitudes quickly develop or existing negative attitudes can quickly become even more entrenched as indicated by negative attitudinal changes and negative item variations.

Item and attitudinal estimate variations

Variations in item estimates from the two administrations of the questionnaire suggested the following changes about PPTs' attitudes in terms of their (1) feelings about, and towards, mathematics; (2) thinking about mathematics and mathematics learning; (3) actions when given problems; (4) personal perceptions and beliefs about primary mathematics and mathematics learning in general; and (5) supportive course feedback. Firstly, positive attitudinal variations by the end of the study suggested PPTs' expressed positive feelings of liking, interest, enjoyment, great satisfaction when individually successful in solving a problem and strong agreement that mathematics was their most favourite subject. Secondly, PPTs thought positively about the utilitarian value and relevance of mathematics skills to daily living and cross-curricular applications. Thirdly, when faced with challenging problems, one of the strategies they would consider included the use of diagrams and/or models to clarify their thinking. Fourthly, positive attitudinal variations demonstrated PPTs' personal beliefs and perceived confidence about their capacity and competence as mathematics learners to use existing knowledge to solve problem/s and a willingness to improve their mathematical understanding. In contrast, negative attitudinal variations suggested entrenched negative attitudes. For example, the decreased item estimates suggested feelings of discomfort, anxiety, nervousness, not needing mathematics to be successful in school and helplessness in learning advanced mathematics even if they tried. As a result, they perceived mathematics learning and problem-solving, as not requiring understanding, but instead, as simply memorizing steps to solve a problem.

In addition to item variations described above, positive attitudinal changes were also noted for the majority of the PPTs that took both preQ and posQ albeit these were quantitatively smaller compared to negative attitudinal changes as shown in Figs. 4 and 5. Such empirical trends are supported by McLeod's (1992) contention that attitudes, whilst they develop with time and experience, once formed are reasonably stable. The results also affirm those by Schoenfeld (1989), McLeod (1992) and Broun et al. (1988) that positive attitudes influence mathematic performance positively whilst negative attitudes influence mathematical performance negatively. This was certainly the case within each category of positive and negative attitudinal changes. It appeared that hardened changes in students' attitudes would have a long-lasting effect and it would probably take more than a semester to cause any significant positive cohort attitudinal changes. However, all of the interviewees and some open questionnaire respondents expressed positive feedback about the impact of the innovative strategies and tools on their thinking and reasoning in mathematics to the extent that they trialled the strategies with their own children and were looking forward to when they would use them with their future primary students. This empirical evidence further supports those of Schoenfeld (1991) and Ernest (1999) about making it more natural for students to think and reason mathematically by deliberately making an effort to change classroom practices and processes.

Potential factors influencing PPTs' attitudes

PPTs' explanations in the questionnaire open-response item and responses from the interviews were analysed for possible factors influencing their mathematics attitudes. Four themes, namely motivational factors, self-improvement, personal perceptions of mathematics and course feedback, were identified from PPTs' explanations of why they would continue taking another mathematics course in the following year. The first two themes collectively demonstrated PPTs' good intentions to persevere and succeed in passing their mathematics courses so that they graduate. Despite the good intentions, the challenges faced, possibly due to the level of difficulty, complexity of learning tasks and/or assessment opportunities, made it difficult to cope with the mathematical and cognitive demands of the FYMM course. These concerns, as raised in their open and interview responses, were categorized under the third theme: personal perceptions of primary mathematics. As some of the Diploma-Early Childhood and Diploma-Special Needs PPTs found out, although the course was initially hard, they persevered until the end to find that they had indeed learnt some new methods and different ways of viewing mathematics to the extent that they tried these new ideas and methods with their own children. Some PPTs even commented with excitement that they looked forward to teaching mathematics in their (future) classes. With the empirical evidence and main findings presented, it was clear that the PPTs were confident and fully cognizant of the demands (both mathematically and cognitively) for teaching mathematics competently. The PPTs were quite keen and willing to continue taking the next compulsory mathematics course in order to be better prepared mathematically to teach and to enable graduation. In terms of the three components of attitudes (cognitive, affective and behavioural) as perceived by some researchers (McLeod 2009; Hogg and Vaughan 2005; Eagly and Chaiken 1993; Weber 1992; LaPiere 1934), it appeared that despite their thinking and cognitive awareness (i.e. beliefs and knowledge about and of the required mathematics for teaching), their behaviour and actions (engagement, achievement and performance) throughout the course have not been sufficiently influenced or changed enough to reflect significantly in their feelings (affective component) about mathematics. Alternatively, the findings may be suggesting that their feelings (love, liking) towards, and their thinking and cognitive awareness of, mathematics need to be substantive and strong enough to influence the way they behave (act, perform and achieve) in mathematics. Although the intent of the course was to introduce innovative strategies and authentic investigations to deliberately change the meaning of PPTs' experiences, the empirical evidence suggests that PPTs were, to various extents, successful (or not) at grasping these intended meanings during the semester with their consequential actions deliberated accordingly to learn (or not to learn) something new. Feelings are thus conceptualized (Gowin 1981) as accompanying any thinking that moves to reorganize meaning as encapsulated by Gowin's notion of 'felt significance' as connection making between feelings and meaningfulness. Furthermore, in an educating context, we are concerned to integrate thinking, feeling and acting (Gowin 1981, p. 42). McLeod (2009) and others (Hogg and Vaughan 2005; Eagly and Chaiken 1993; Weber 1992; LaPiere 1934) also found out that the kind of experience a learner gains through his/her interaction with the environment is key to developing initial perceptions and more long-lasting attitudes, and it is these, in their view, that will determine learning behaviour. That it seemed to take longer than a semester for a number of students to develop positive mathematics attitudes was evident from the data presented in this article.

The third theme, perceptions and personal beliefs of the mathematics content, found that prospective teacher education content and the 'taught' mathematics PPTs observed in primary classrooms during teaching practicum highlighted an area that has been vigorously researched (e.g. Ball 1990; Ball and Bass 2000; Hill et al. 2004; Ball et al. 2008). Recently, Ball et al. (2008; Hill et al. 2008) recommended that mathematics should be content-specific and more meaningful to the knowledge used in teaching children. They also further defined specialized content knowledge for teaching as the mathematics knowledge that allows the teachers to follow students' mathematical thinking, evaluating the validity of student-generated strategies and making sense of a range of student-generated solution paths (Hill et al. 2008, p. 377). These skills are embedded in the new 2013 Curriculum to be strategically developed in primary students, as well as explicitly taught, demonstrated and practiced during the FYMM course, through the use of working mathematically processes, mental computation and multiple problem-solving strategies of mathematical tasks and investigations.

The attitudinal data presented suggested that the most influential factor of the PPTs' mathematics attitudes was based upon whether the learning experiences with the FYMM course's various learning activities and assessment tasks were successful and continuous. As the post-questionnaires were administered after teaching practicum, some PPTs used the opportunity to voice their perceptions and personal beliefs regarding the FYMM course based on their encounters with the 'taught' curriculum in primary classrooms. Be that as it may, PPTs' perceptions would need to be put in perspective within the larger context of (1) existing evidence from SPELL I and SPELL II national results (SMESC 2008), (2) empirical findings from Afamasaga-Fuata'i and her colleagues' studies (2007a, b, 2008, 2010) of foundation students' and prospective teachers' mathematical competence and (3) SMESC's implementation plan for the new 2013 Mathematics Curriculum.

The teaching approach and deliberate selection of activities, authentic tasks and innovative tools introduced and used throughout the course was an attempt to encourage students to think and reason mathematically 'outside of the box' in ways that were different and unlike those (i.e. traditional approach) that they have been exposed to in their previous mathematics classes. As aforementioned, a national need exists to offer innovative, constructivist and socioculturally driven learning programmes to target the development of students' critical thinking, reasoning and analytical skills whilst engaged in working and communicating mathematically to solve mathematics problems that are authentic, interesting and challenging, a philosophical and conceptual approach that is currently being promoted and is underpinning the new Curriculum. Vitally important is the fact that the new Curriculum will be in primary schools by the time the PPTs graduate from NUS. To date, both national evidence and empirical evidence corroborate the importance of explicitly developing students' numeracy and mathematics knowledge and skills in solving word problems and engaging students with authentic mathematical investigations/activities. Consequently, the new Curriculum has been reinvigorated to promote a more sociocultural pedagogical approach in its teaching and learning of mathematics with a focus on the needs of students instead of the very traditional transmission teaching–learning model that is commonly practiced in many Samoan primary classrooms (SMESC 1995). Further highlighted by the fourth theme from PPTs' open responses is another concern regarding the need for upgrading professional learning and development opportunities for existing, practicing teachers to familiarize themselves with the new Curriculum. SMESC has begun to address this need with ongoing teacher workshops at the time of writing this article.

Finally, PPTs' perceptions of the FYMM content also provided constructive feedback to appropriately revise and rearrange the course structure and sequencing of learning experiences and assessment opportunities to optimize future PPTs' learning experiences and skills development in better meeting the demands of the new Curriculum. Another issue, as also previously identified elsewhere (Afamasaga-Fuata'i et al. 2007a, b, 2008, 2010), is the need to address PPTs' level of mathematics competence before entering the FYMM course. Doing so would ensure that PPTs are more focused on developing and practicing a more sociocultural approach to learning and teaching within the teaching space of a semester before going out on their teaching practicum. Furthermore, PPTs' comments may be interpreted as indicative of some resentment or perhaps frustrations that the practiced mathematics teaching and learning approach of the FYMM course was too dissimilar to the mathematics and teaching style PPTs encountered during teaching practicum. This plausible interpretation highlighted another curricular concern, namely the need to closely monitor primary classroom practices to ensure that the 'taught' curriculum aligns or complies with the standards of the new mathematics curriculum.

By the end of the study, the main findings included the overall cohort's mathematics attitudes and more detailed trends of paired PPTs' attitudinal estimate changes and the nature of variations in item estimates. Whilst the overall cohort attitudinal trend was negative, a more elaborative view was provided by examining the variations in item and PPTs' paired attitudinal estimates. Both the positive and negative variations, including PPTs' open and interview responses, suggested that learning experiences in the FYMM course influenced general perceptions about mathematics learning and therefore, one's actions when confronted with problems to solve. These findings contribute to the mathematics education literature on attitudes and innovation in general and in particular to the scarce literature in mathematics teaching and learning in Samoan educational institutions.

Implications

The main findings of the study have implications for teacher education and ongoing professional learning and development programmes universally, in particular attempting to shift PPTs' and teachers' thinking and feeling about mathematics, their actions when confronted with problem-solving and perceptions about the nature of mathematics teaching and learning that is more sociocultural and student-focused. That this new pedagogical approach requires: the ongoing implementation/application of working mathematically processes, mental computations and multiple problem-solving strategies as students engage with mathematical investigations of authentic contexts and solve mathematical tasks with or without the support of metacognitive tools, is both a philosophical and conceptual mind shift for prospective and practicing teachers to recognize, acknowledge and accept as an integral part of the new mathematics curriculum. The main findings from our study provided empirical evidence of some areas such as sufficient provision of time for both teachers and students to become familiar in applying and practicing the new approach before being able to work effectively and efficiently with it. This is particularly crucial when designing viable models to guide future professional development and learning activities with practicing teachers in addition to current reforms in the prospective teacher education programme. When appropriately executed, it can contribute substantively to a more productive curriculum implementation in Samoan classrooms. Further research is needed to accompany and track the implementation of the new mathematics curriculum in Samoan classrooms.

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Appendix 1: Tasks 1, 2 and 3 descriptions

Task 1: Authentic Mathematical Investigations: [Work in PAIRS]

Choose an authentic context from the following:
1. Sustainable Development – Focus: Water (environment or climate)
2. Samoan Fale
3. Other Samoan Artifacts or Practices

Each investigation must include two types of methods: (a) an Estimation Method and (b) an Accurate Method and (c) Error Percentage between Accurate Answer and Estimated Answer.

Part 1

Questions to ask of your authentic context:

a. What mathematics is involved in this real life context?

b. What mathematical ideas can be used to create an investigation out of this context?

c. Develop a focus question or focus questions for your investigation.

d. Construct a concept map of the mathematical ideas identified in (a) and (b). Your concept map should include about **25-30 concepts**, arranged in a **hierarchy from most general to most specific**, with **connecting links between concepts** including **descriptive phrases on the links** describing the nature of the relationship between the first and second concepts.

e. Identify the **data** you are going to collect so that you can answer your focus question(s).

f. Develop a **plan** for collecting it. **Your Plan** should have the sections: (i) **Introduction** – general description of your authentic context (ii) **Mathematical Analysis** - List of key mathematical ideas and concepts based on your answers to questions (a) and (b) above, (iii) **Focus Questions** – questions that will guide your investigation and for which answers are to be obtained during the semester, and (iv) **Data to be collected** – describe what data you will collect, and (v) **Significance of Investigation** - describe why your investigation is important to learning mathematics in primary schools in terms of linkages to content (e.g., topics) and processes to enable working mathematically and problem solving in primary schools. (**Your Plan should be no more than two-A4 pages**).

Part 2

a. **Report of Mathematical Investigation -** Analyse your data and discuss your findings to answer FQs in Part 1 as a report. **The Report** should include the sections: (i) **Introduction** – general description of your authentic context (ii) **Focus Questions** – questions that guided your investigation (iii) (iv) **Data Collected** – describe the data you collected, and present raw data in a table and (v) **Results and Discussion** – (a) describe how you re-organised your raw data (must include two types of data display) ready for analysis, (b) describe how you analysed the data collected, interpret your displayed data, and discuss interesting patterns (vi) **Conclusions** – formulate answers to your focus questions based on your data analysis and interpretations (vii) **Recommendations -** describe how your findings can be used to improve the meaningful learning of mathematics in primary schools in terms of (a) linkages to content (e.g., topics), (b) linkages to working mathematically and problem solving processes (see handouts given out in class for a list of these WM and PS processes/strategies). *Your report should be about four-A4 pages.*

b. **Extension of Activity to Link to TWO other Topic Area(s) –** create and/or modify your activity to extend the investigation so that it links to TWO other, different topic areas in primary mathematics. *Your extended activity should be described in about one-A4 page.*

c. **Concept Map of your Mathematical Investigation (CMMI)** – Bring together your mathematical ideas (from Part 1) and those that emerged from your data analysis and findings to form an expanded list of key mathematical ideas and concepts. Then construct an **expanded CCMI of 35-40 concepts including illustrative examples**. *Use one-A3 paper or two-A4 papers glued together to give you a bit more space.*

d. Construct a vee diagram of your mathematical investigation using brief descriptions as appropriate for the relevant sections: (i) Activity, (ii) What are the questions I need to answer? (iii) What are the important ideas? (iv) What is the data I collected? (v) What do I know already? (vi) How do I find my answers? (vii) What are the most useful things I learnt?, (vii) What do I want to study next? and (viii) What is Mathematics? *Use one-A3 paper or two-A4 papers glued together.*

Task: 2: Vee Diagrams of Math Problems and their associated Concept Maps [INDIVIDUAL work]

This assignment is in two Parts.

Part 1

Choose any 2 mathematics problems or activity:

a. Complete a **vee diagram** for each problem or activity by completing the sections:

i. Problem/Activity,
ii. What is the question I need to answer?
iii. What are the important ideas?
iv. What is the given information?
v. What do I know already?
vi. How do I find my answers?
vii. What are the most useful things I learnt? Or What do I want to study next?
viii. What is Mathematics?

b. Use the information displayed on the left-side of the vee diagram (i.e. conceptual side) to construct a concept map of the mathematics used in solving the problem. Your concept map should include about **15 – 20 concept ovals**, arranged in a **hierarchy from most general to most specific**, with **links connecting concepts** including **descriptive phrases on the links** describing the nature of the relationship between concepts.

c. **Reflections** – (about 1-page) –

i. Describe how constructing the vee diagram and concept map may have helped or not helped your understanding of the mathematics used in the problems.

ii. Comment on any discoveries (relationships between mathematical ideas) you may have made as a result of doing this assignment.

iii. Suggest how you might use what you learnt from this assignment in teaching primary mathematics.

Part 2

Choose any 2 mathematics problems or activities **that are DIFFERENT from those done for Part 1**:

a. Complete a vee diagram for each problem or activity by completing the sections as described for Task 2 Part 1, section a.

b. Show a **second method** of solving the same problem in (a). Include this second method and relevant prior knowledge on the same vee diagram as in (a)

c. Use the information displayed on the left-side of the vee diagram (which justifies Methods 1 and 2 from [a] and [b] above) to construct a concept map of the mathematics used in solving the problem.

d. **Reflections** – (about 1-page) –

i. Describe how constructing the vee diagram and concept map may have helped or not helped your understanding of the mathematics use in the problem.

ii. Comment on any discoveries (relationships between mathematical ideas) you may have made as a result of doing this assignment.

iii. Suggest how you might use what you learnt from this assignment in teaching primary mathematics.

Task 3: Content–Curriculum Links (2 parts) [work in GROUPS OF 2 OR 3]

You will be assigned: (1) a **Sub-Strand Topic Learning Outcome** (LO) and (2) a sequence of **Knowledge & Skills (K&S) LO** (either Years 1 to 3 [Sequence A] or Years 3/4 to 6 [Sequence B]) *including* its relevant **Working Mathematically (WM) LO** from the draft primary curriculum to analyse and to identify its relevant content in terms of main ideas, knowledge and skills to be taught within each year level (Referenced: Analysis).

Part 1

a. Identify which is *New Knowledge (NK)* to be taught for that Year Level and which is *Prior Knowledge (PK)* upon which NK is to be developed from. Also identify recommended and/or potential *Links to other Topics or subjects (LTO)* as recommended in the draft curriculum (Referenced: Labeling).

b. Use your findings from your Analysis and Labeling exercises above, to construct *the first draft of your* **Topic Concept Map** (TCM) (i.e., your Topic is based on your Sub-Strand Topic LO in [1] above) for your assigned sequence, either Sequence A or Sequence B as described above in (2). *Your draft TCM should have **at least 40 concept ovals.***

Notes (1) You will be assigned either a **Sequence A** or **Sequence B** not both unless it is with approval from us.
(2) use different colours for the concept ovals to differentiate between NK, PK and LTO

Part 2

a. Revise your Topic Concept Map (TCM) based on feedback from your marked Task 3 Part 1 assignment.

b. *Expand* your revised TCM in (a) to make it *more comprehensive* by including (i) more relevant concepts that you may have left out in the draft based on further consideration of your assigned sequence and K&S LO, and (ii) more examples (briefly described in 2 or 3 words) as suggested by the associated WM LO of your K&S LO.

Note: *Your comprehensive TCM should have **at least 60 concept ovals** and you should also include **illustrative examples** of concepts where appropriate*. **Every** connecting link should have a *descriptive phrase* describing the nature of the inter-connection between the beginning concept (outgoing link) and end concept (incoming link).

Below is a quick count of K&S LO and WM LO for each Sub-strand to give you an idea of the number of LO you will be expected to analyse as described above.

Sub-Strand LO	**Total# K&S LO**	Y1-3 K&S LO	Y4-6 K&S LO	#of WM LO	Sub-Strand LO	**Total # K&S LO**	Y1-3 K&S LO	Y4-6 K&S LO	# WM LO
NR1.1-NR6.1	**66**	49	17	49	MS1.1– MS6.1	**57**	28	29	32
NR1.2–NR6.2	**41**	29	12	55	MS1.2– MS6.2	**39**	22	17	35
NR1.3– R6.3	**54**	31	23	53	MS1.3–MS6.3	**53**	27	26	38
NR1.4–NR6.4	**45**	16	29	43	MS1.4– S6.4	**38**	23	15	27
NR1.5–NR6.5	**29**	16	13	24	MS1.5– MS6.5	**47**	36	11	47
PA1.1aPA6.1a	**22**	14	8	43	SG1.1–SG6.1	**47**	23	24	29
PA1.1b– 6.1b	**31**	14	17	13	SG1.2–SG6.2	**84**	38	46	64
DA1.1–DA6.1	**39**	18	21	40	SG1.3–SG6.3	**50**	22	28	18

Appendix 2: Items in the attitudinal questionnaire

1. Mathematics is very interesting to me and I enjoy my mathematics classes.
2. When doing mathematics, my mind goes blank, and I am not able to think clearly.
3. I like solving mathematics problems.
4. Mathematics makes me feel uncomfortable and impatient.
5. I have always enjoyed studying mathematics in school.
6. I am nervous in mathematics classes because I feel I cannot do mathematics.
7. It makes me nervous to even think about having solving a mathematics problem.
8. I really like mathematics; it's enjoyable.
9. I can cope with a new problem because I am good in mathematics.
10. I get worried when solving a problem that is different from the ones done in class.
11. I can find many different ways of solving a particular mathematics problem.

12. Most of the time, I need help from the teacher before I can solve a problem.
13. I believe that if I use what I know already, I can solve any mathematics problem.
14. I have forgotten many of the mathematical concepts that I have learnt in previous mathematics classes.
15. I learn mathematics by understanding the main ideas, not by memorizing the rules and steps in a procedure.
16. If I cannot solve a mathematics problem, I just ignore it.
17. Successfully solving a problem on my own provides satisfaction similar to winning a game.
18. I feel nervous when doing mathematics.
19. My most favourite subject is mathematics.
20. Mathematics classes provide the opportunity to learn skills that are useful in daily living.
21. To succeed in school, you don't need to be good in mathematics.
22. Mathematics is not my strength and I avoid it whenever I can.
23. I don't think I could learn advanced mathematics, even if I really tried.
24. Doing mathematics encourages me to think creatively.
25. I learn to think more clearly in mathematics if I make a model or draw diagrams of the problem.
26. Mathematics is important for most jobs and careers.
27. Solving mathematics problems helps me learn to think and reason better.
28. To succeed in life you need to be able to do mathematics.
29. Mathematics is needed in understanding newspaper reports and finance graphs.
30. Communicating with other students helps me have a better attitude towards mathematics.
31. I am interested and willing to improve my understanding of mathematics.
32. The skills I learn in mathematics will help me in other subjects at school.
33. I do not have to understand mathematics, I simply memorize the steps to solve a problem.
34. I learn mathematics well if I understand the reasons behind the methods used.
35. I intend to continue taking mathematics next year.

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